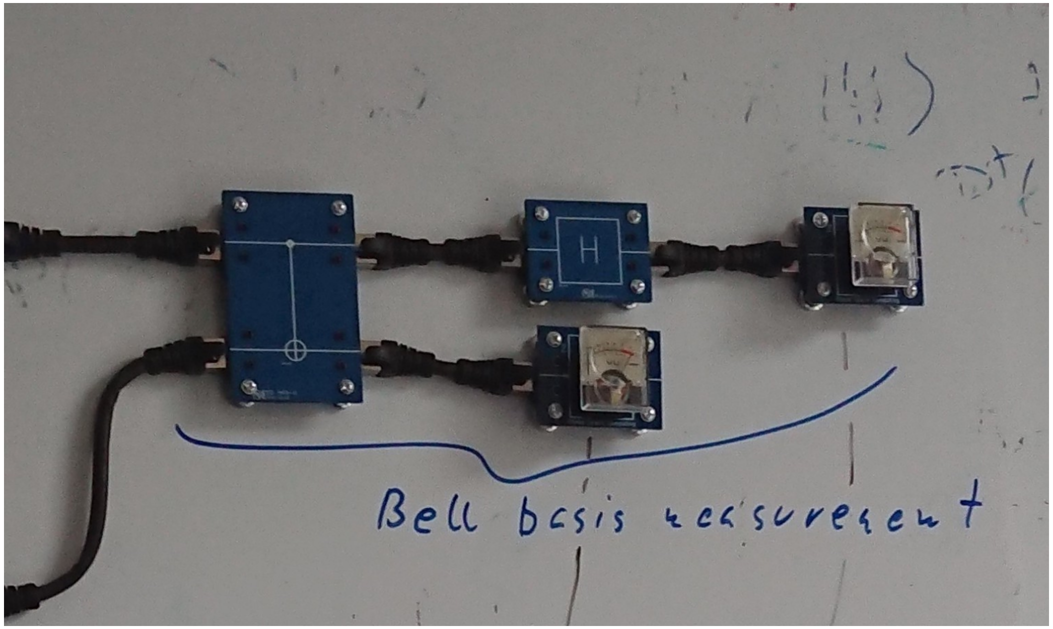
Is entanglement inevitable to process quantum information?

On classical simulation of quantum protocols



Jakub Czartowski, Karol Życzkowski, UJ

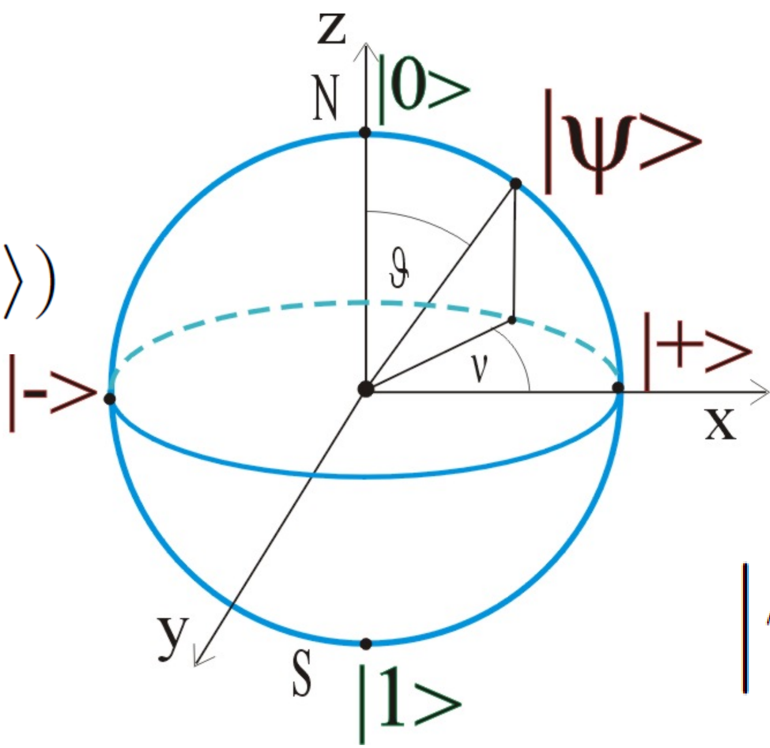
AGH, Cracow, April 16 2019

Simple system – quantum states of a single qubit:

Classical states : |0>, |1>

Non - classical : |+>, |->, |ψ>

$$\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$



$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$|\psi'\rangle = U|\psi\rangle$$

Quantum superposition – a pure state $|\psi\rangle$

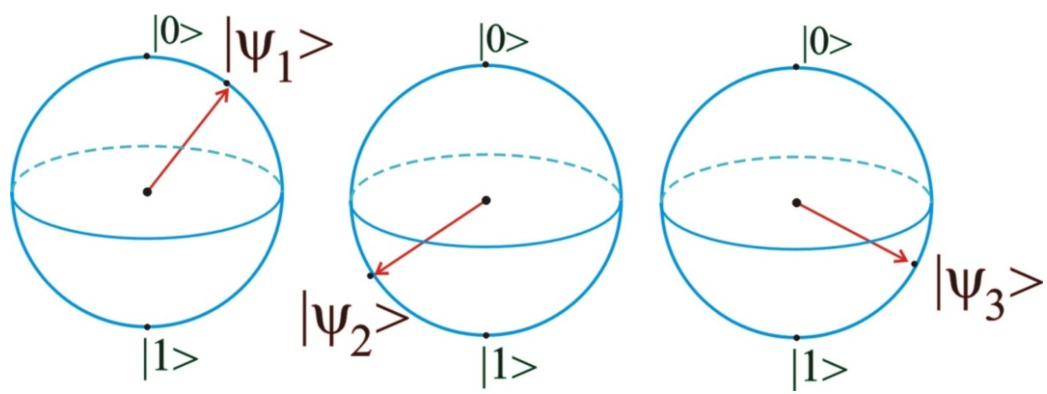
differs from a classical mixture of quantum states

(a point inside the Bloch ball)

Theory of classical information works with bits

Theory of quantum information relies on qubits:

$$|\psi_1\rangle, |\psi_2\rangle, |\psi_3\rangle \ldots$$



Advantages:

a) Larger space of states allowed

b) Larger set of operations available

Bipartite systems : {A,B}

a) **Separable** (product) state - no correlations !

$$|\psi_{\text{sep}}\rangle = |\psi_A, \psi_B\rangle = |\psi_A\rangle \otimes |\psi_B\rangle$$

b) **Entangled state** (not product) – shows correlations

$$|\psi_{\text{ent}}\rangle \neq |\psi_A, \psi_B\rangle = |\psi_A\rangle \otimes |\psi_B\rangle$$

Superposition of two bipartite states

example: the **Bell state**

$$\boxed{|\psi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)}$$

Entangled state reveals quantum correlations present due to previous interactions between subsystems

Quantum information processing

makes use of :

superposition of states $$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

and entanglement $$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

Applications:

* quantum teleportation
* quantum computing
* quantum cryptography

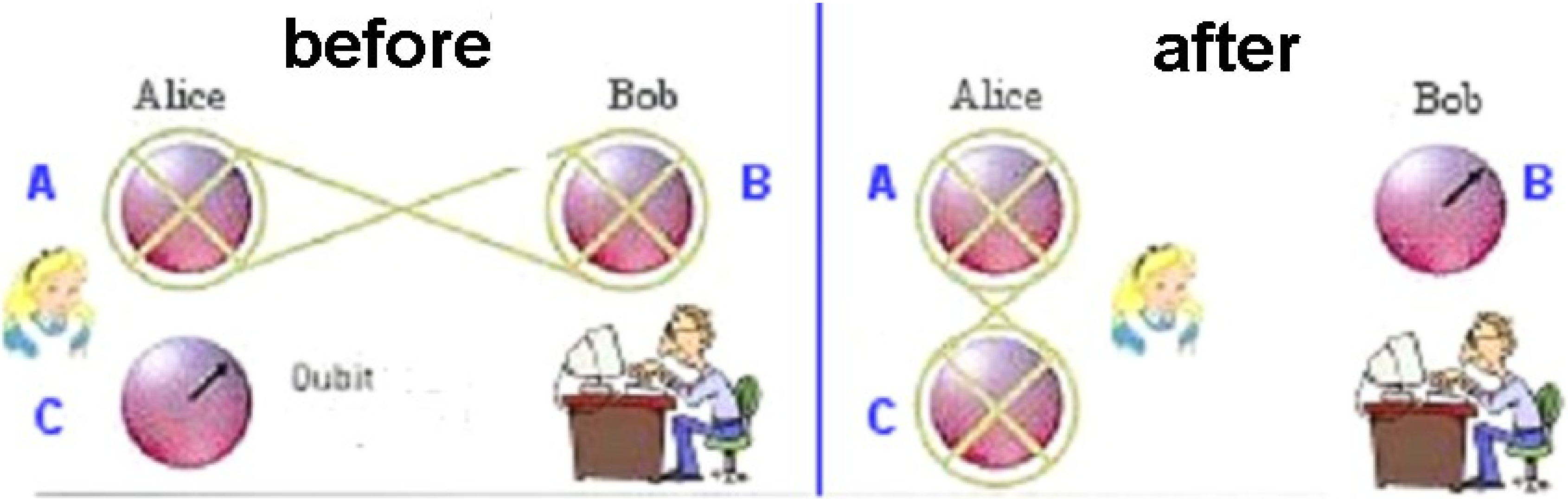
quantum metrology, simmulations

quantum games, finances

quantum machine learning !

… … …

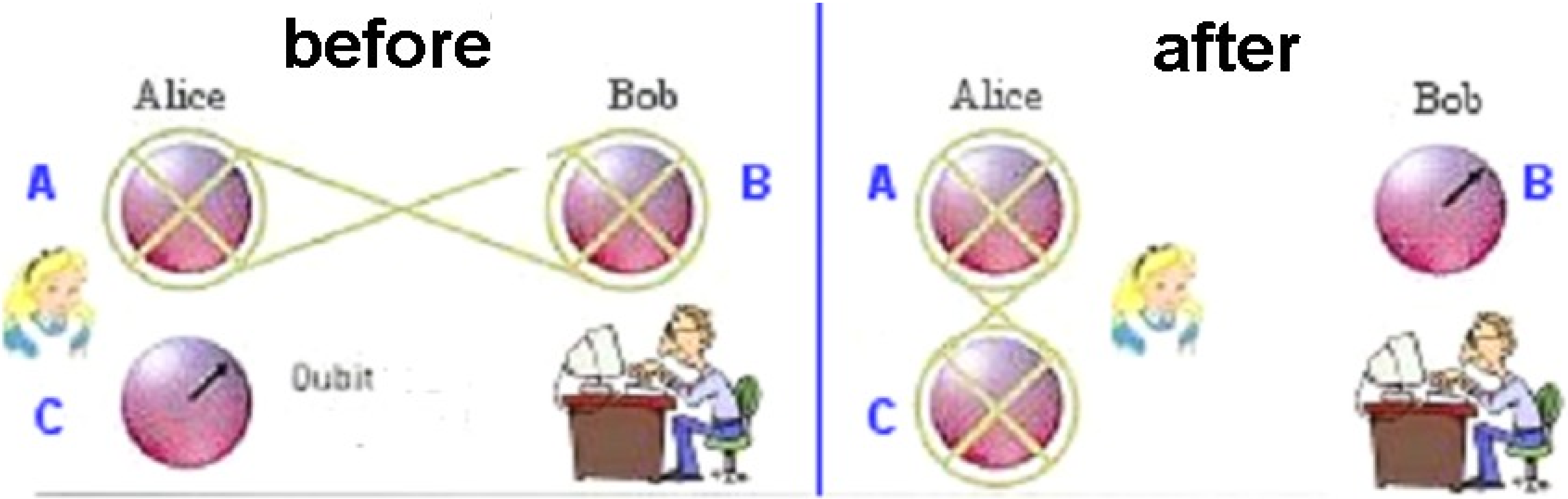
Scheme of quantum teleportation

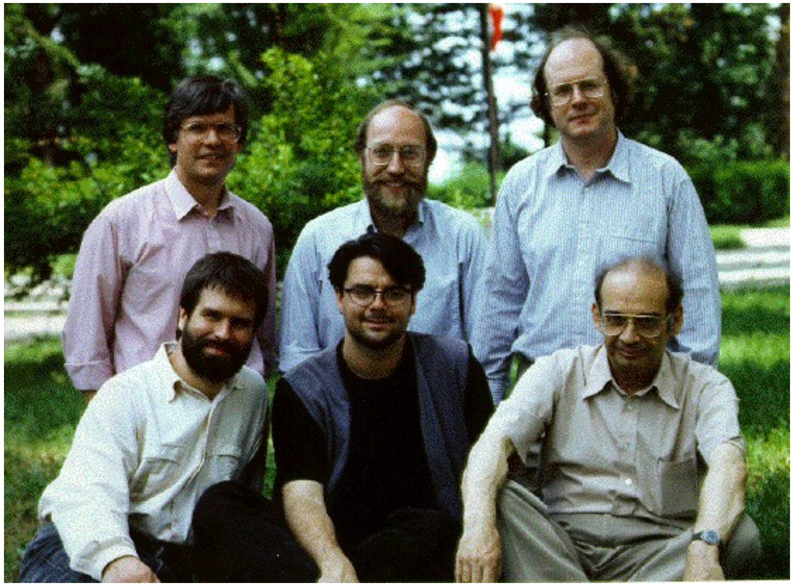


Making use of a Bell state
Alice sends her unknown
state C reconstructed by Bob

What is teleported ?

Scheme of quantum teleportation





Bennett et al. 1993

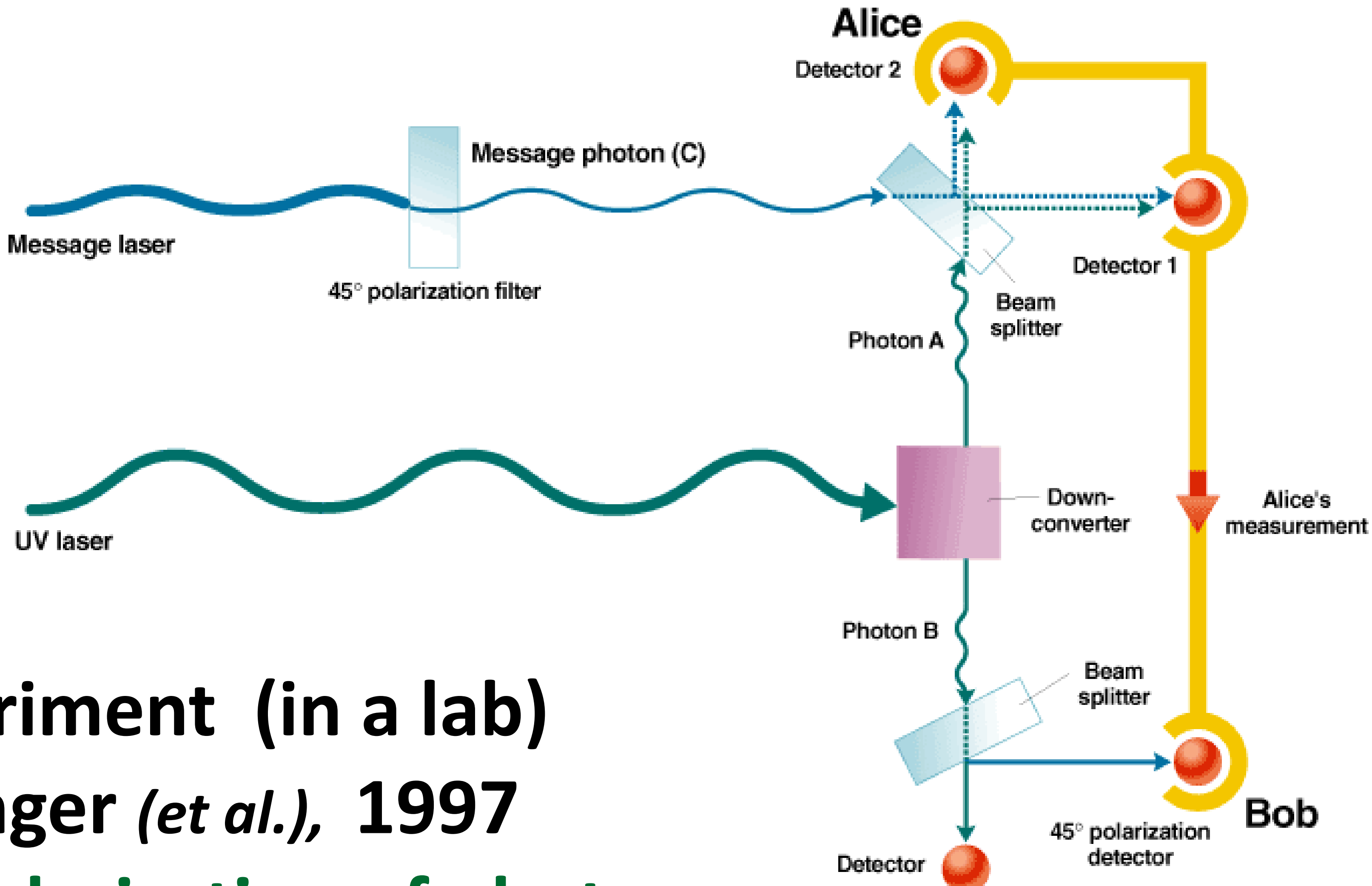
Making use of a Bell state
Alice sends her unknown
state C reconstructed by Bob

What is teleported ?

Quantum information !

quantum teleportation

theory: Bennett et. al. 1993

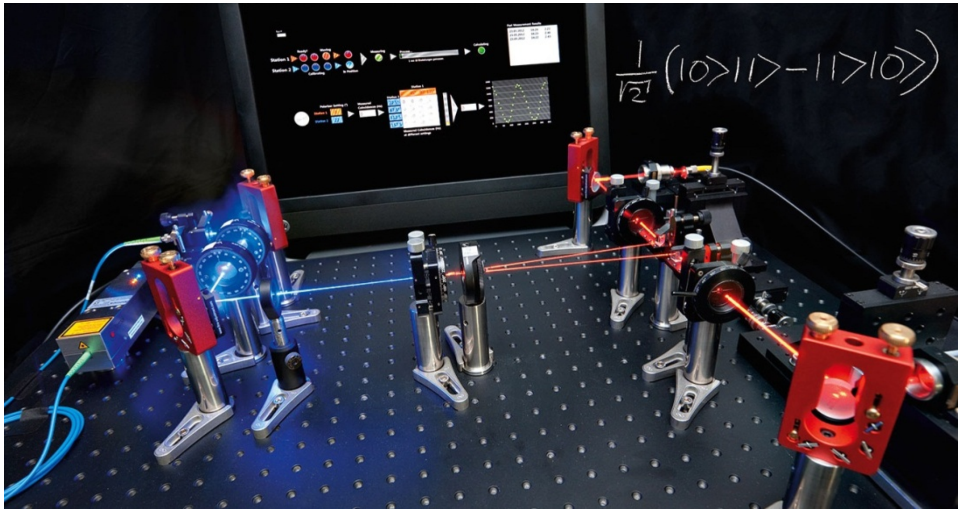


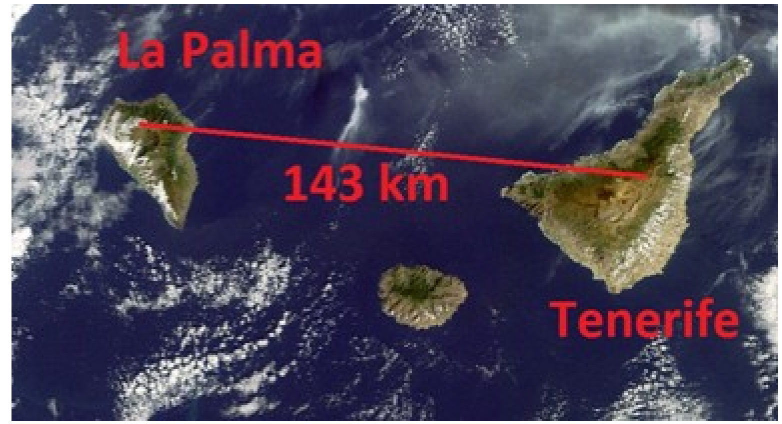
experiment (in a lab)

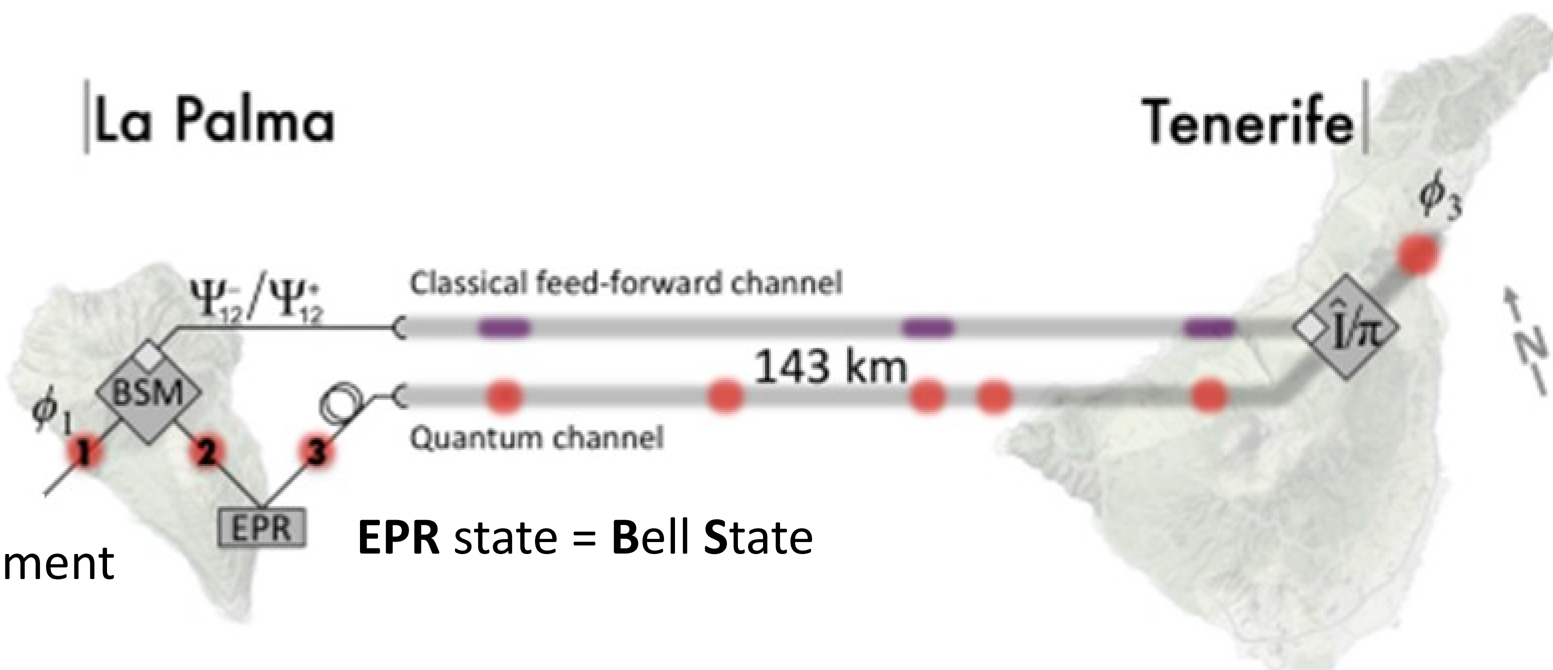
Zeilinger *(et al.),* 1997

polarization of photons

quantum teleportation at 143 km, Canary Islands
European Space Agency, 2012



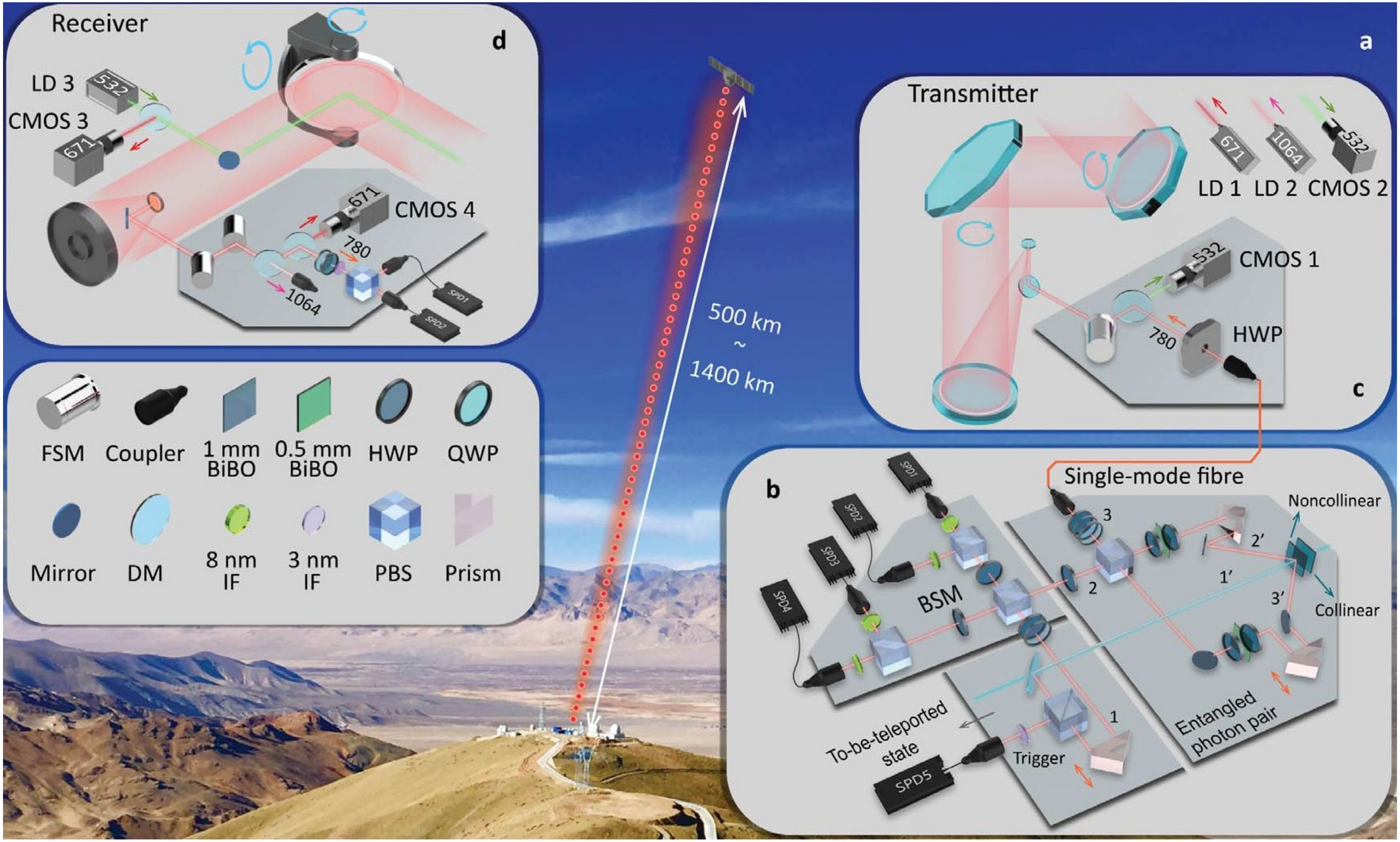




Bell
State
Measurement

EPR state = **B**ell **S**tate

Ground – to satellite quantum teleportation at 1400 km



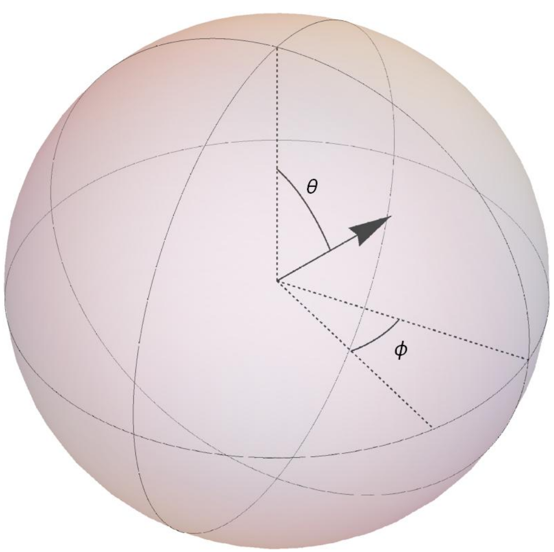
group of **J.-W. Pan,** *Nature* **2017**

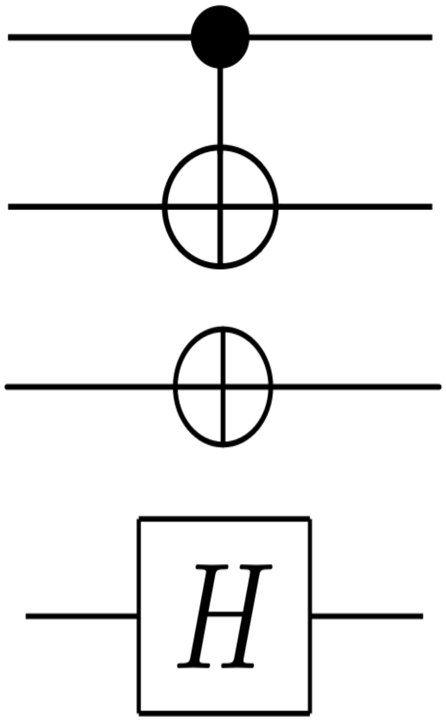
Basic ingredients for quantum computation:

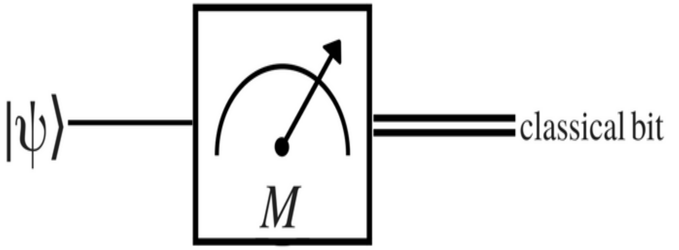
- **Quantum states,**
- e.g. qubits

- **Quantum gates:** unitary operators

- **Measure-**
- **ment**





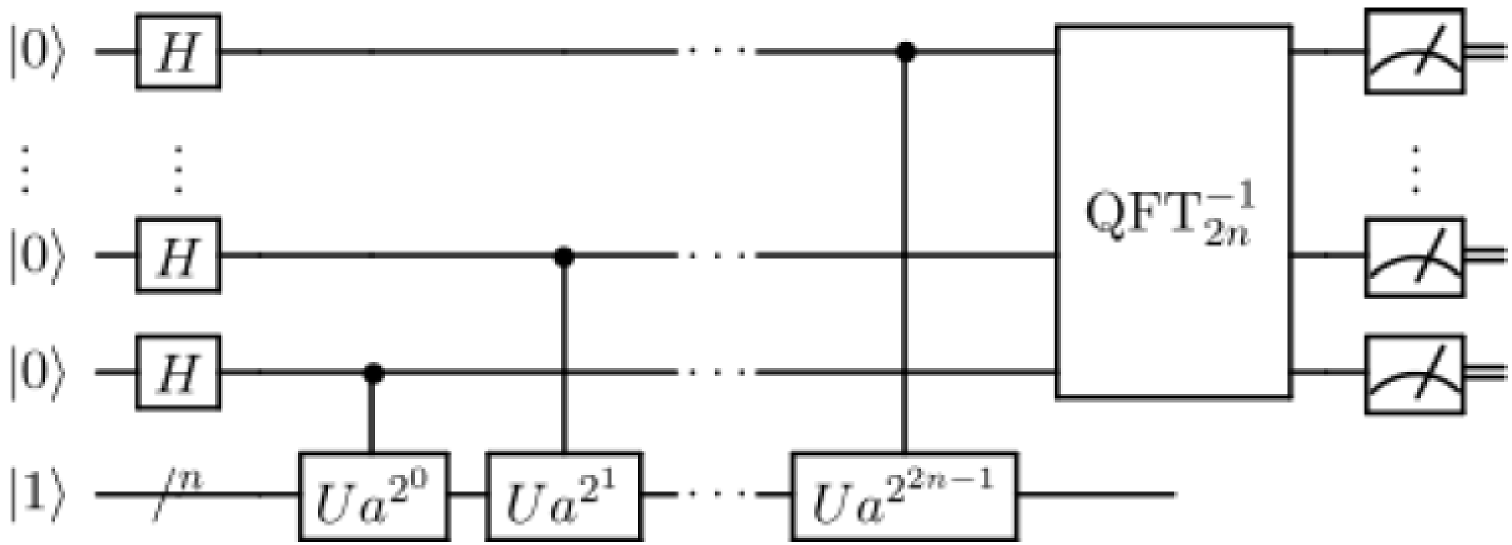


Quantum computing: Shor's algorithm (1994)

Factorization of a number M is as difficult as finding the period T of a periodic function, $f(x)= f(x+T)$

Quantum Fourier transform (QFT)

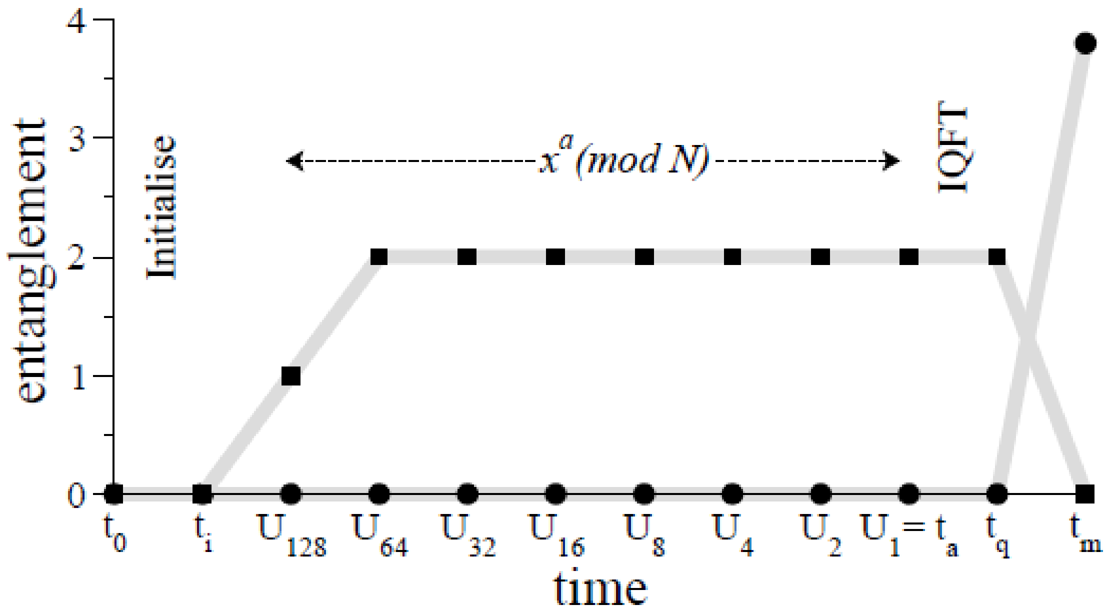
a unitary operation acting on n qubits with $M < 2^n$



Complexity measured by number of operations :

classical ~ $\exp(M^{1/3})$ **quantum ~ $M^3 \log M$**

Pure-state realization of Shor's algorithm relies on quantum entanglement



time evolution of entanglement entropy between registers (Shor's algorithm for factoring 15),

Kendon & Munro 2006

However, entanglement used does not correlate with the number factorised!

formalism of **density matrices** – so called **mixed states**

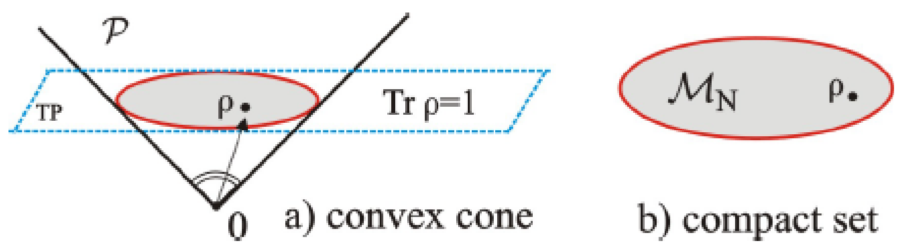
Set $\mathcal{M}_N$ of all mixed states of size N

$$\mathcal{M}_N := \{\rho : \mathcal{H}_N \to \mathcal{H}_N; \rho = \rho^\dagger, \rho \geq 0, \mathrm{Tr}\rho = 1\}$$

example: $\mathcal{M}_2 = B_3 \subset \mathbb{R}^3$ - Bloch ball with all pure states at the boundary

The set $\mathcal{M}_N$ is compact and **convex**: $\rho = \sum_i a_i |\psi_i\rangle\langle\psi_i|$, where $a_i \geq 0$ and $\sum_i a_i = 1$.

It has $N^2 - 1$ real dimensions, $\mathcal{M}_N \subset \mathbb{R}^{N^2-1}$.



a) convex cone b) compact set

Entanglement of mixed quantum states

Mixed states of a bi-partite system, (A, B)

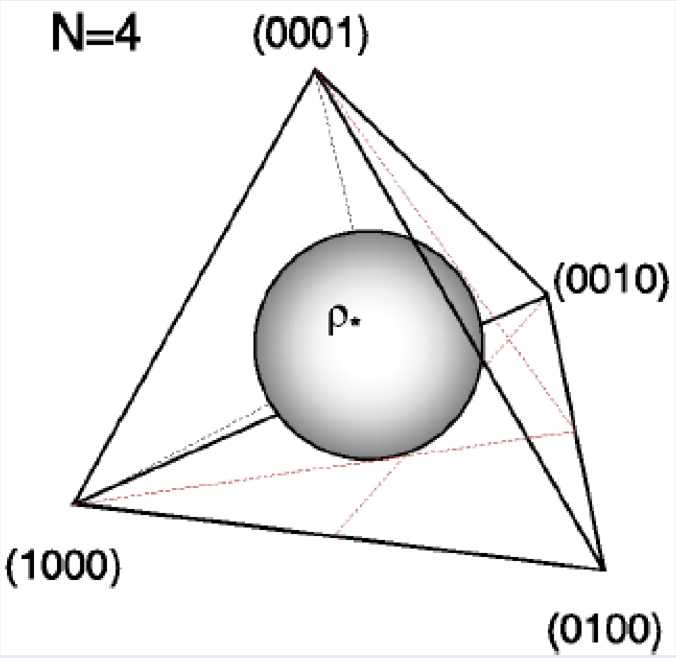
- **separable mixed states:** $\rho_{\mathrm{sep}} = \sum_j p_j \rho_j^A \otimes \rho_j^B \quad (**)$
- **entangled mixed states:** all states not of the above product form.

How the set of separable / entangled states looks like?

Two–qubit mixed states

The maximal ball inscribed into $\mathcal{M}^{(4)}$ of radius $r_4 = 1/\sqrt{12}$ centred at $\rho_* = \mathbb{1}/4$ is separable !

Central ball is separable



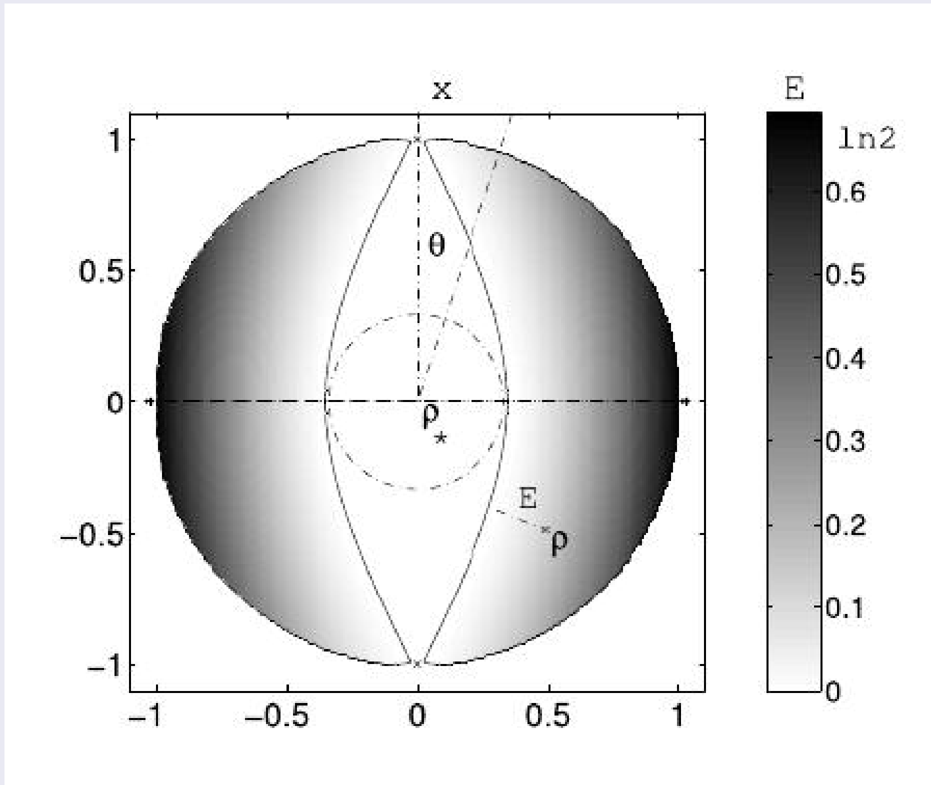
thetrahedron of eigenvalues

K. Ż, P. Horodecki, M. Lewenstein, A. Sanpera, 1998

How to quantify entanglement?

Two–qubit mixed states

Degree of entanglement: a **distance** to the closest **separable state**

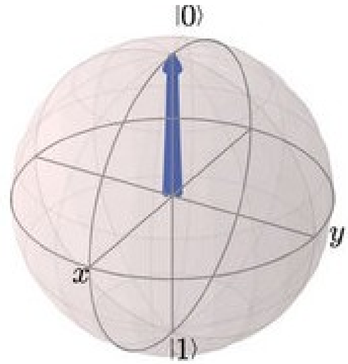


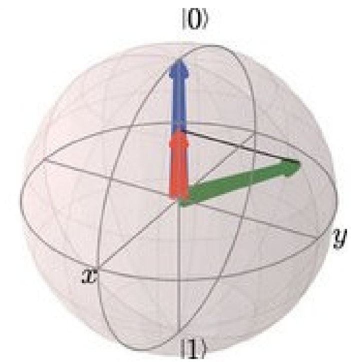
Central ball is separable

(E = **Entanglement** of formation)

K. Ż, M. Kuś, 2001

NMR quantum computing deals with mixed states



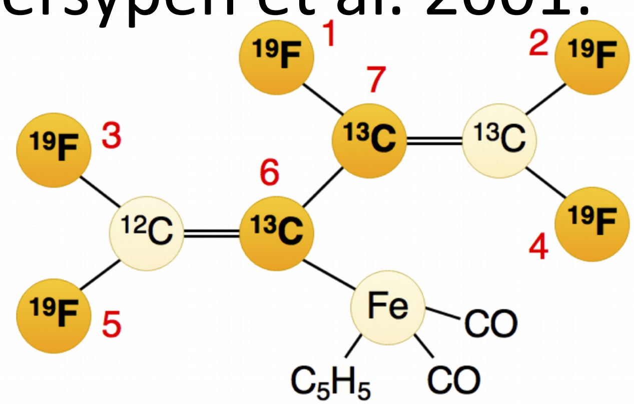


pure state, $|\psi\rangle\langle\psi|$
(at the Bloch sphere)

pseudo-pure state (shorter Bloch vector), $\rho=1/N + a|\psi\rangle\langle\psi|$

NMR realisation of Shor's algorithm

Vandersypen et al. 2001:



7-qubit molecule used

Uses states of a small purity which are separable!

No **entanglement** is used at all to factorize 15 = 3 x 5...

Contemporary implementations of Quantum Computing

Implementation method	Advantages	Disadvantages
NMR-based	**Good decoherence time** **Room temperature operation**	Slow gates Poor scalability
Ion traps	Scalability	Slow gates Limitations on concurrent operations
Super-conducting circuits	**Fast gates** Advanced experimental demonstration	Low coherence time High sensitivity to background magnetic field

And others…

Some of proposed resources used in Quantum Computing

- Interference
- **Entanglement**
- Nonlocality
- Contextuality

Can we simulate some of it with classical systems?

Inspired by the *Toy Model* of **Rob Spekkens** (2004),
Niklas Johansson and **Jan-Åke Larsson** (2017)
designed **Quantum Simulation Logic**

Quantum simulation logic [QSL]

Quantum Inf Process (2017) 16:233
DOI 10.1007/s11128-017-1679-7

CrossMark

Efficient classical simulation of the Deutsch–Jozsa and Simon's algorithms

Niklas Johansson[1] · Jan-Åke Larsson[1]

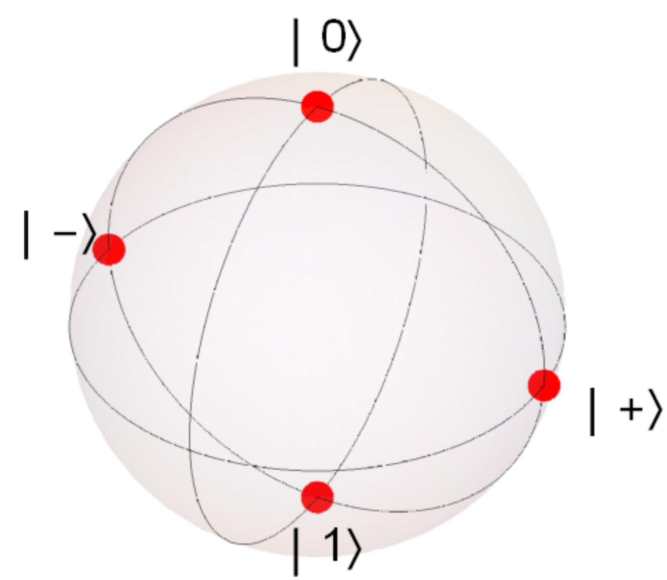
key idea:

Apart of two classical states,

|0> and |1>

mimic also two quantum states,

|+> and |->



States – preparation and available space

COMPUTATIONAL BIT

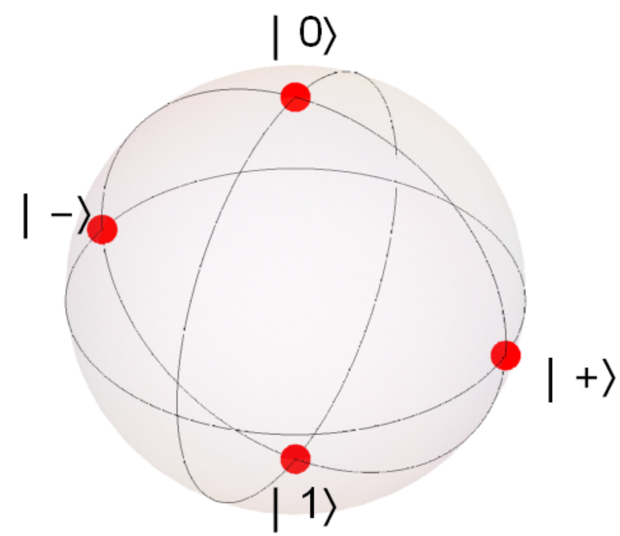
Classical bit set to a known value

0 or 1

PHASE BIT

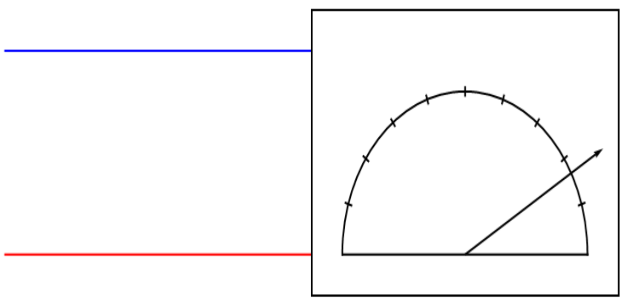
Bit selected randomly from uniform distribution

- States in QSL consist of two bits, computational and phase bit.
- This gives a total of 4 available states.
- These correspond to North, South,
- „West” and „East” poles of the Bloch sphere.

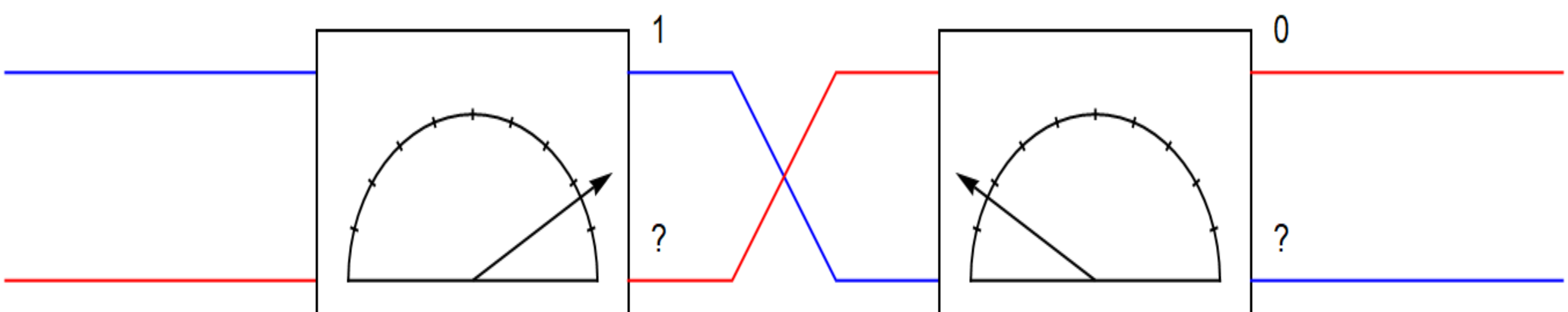


Measurement of a state

- Measurement in QSL framework simply returns value of the **computational bit**



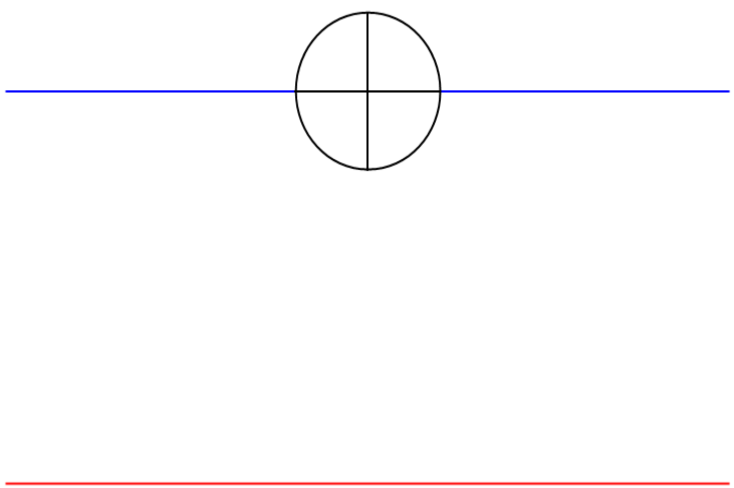
In order to be consistent with requirement that we can know only one bit of information at a time, the **phase bit** needs to be randomized, $p_0=p_1=1/2$

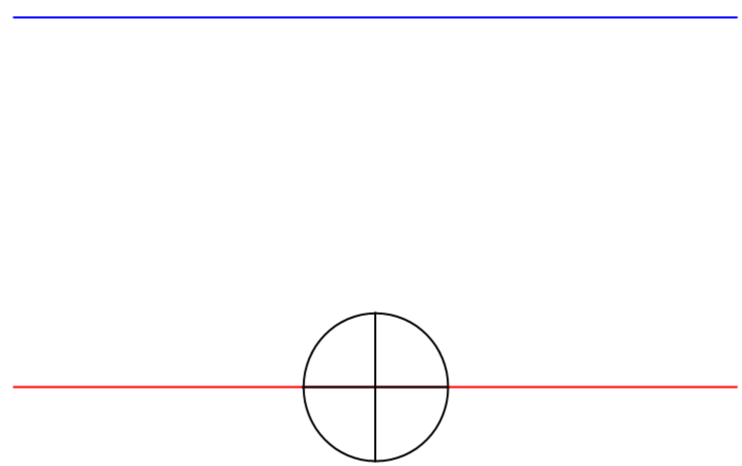


One-qubit gates: **X** and **Z**:

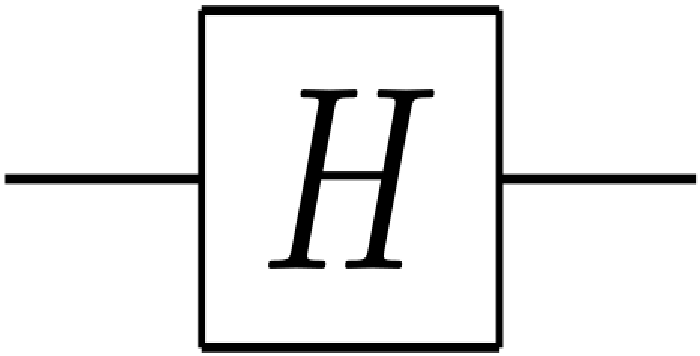
$$\sigma_x = NOT = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

=negation of **blue** or **red** bit

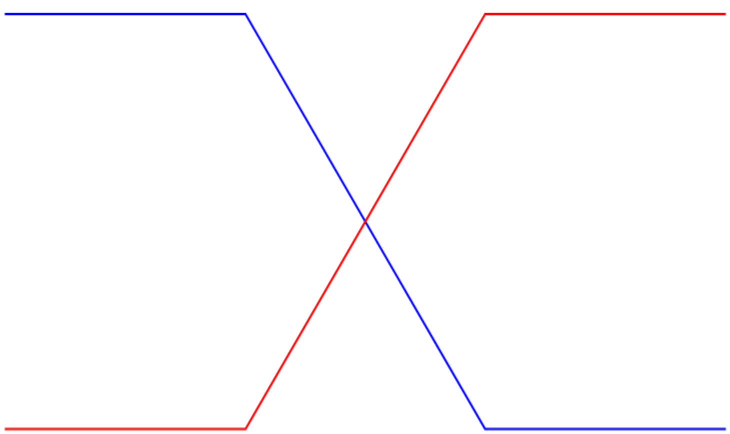




Hadamard gate



$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$



= swap the wires:

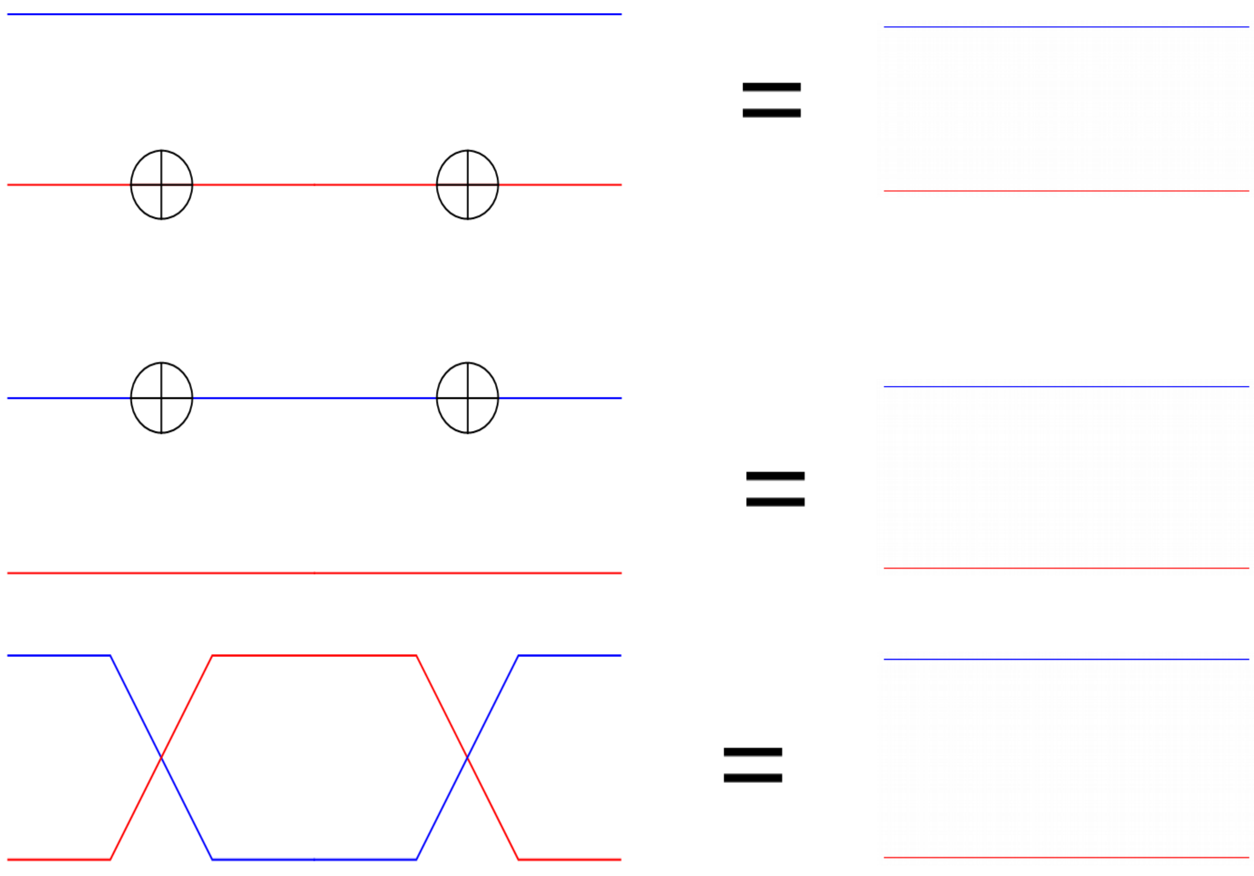
H|0>=|+>

and

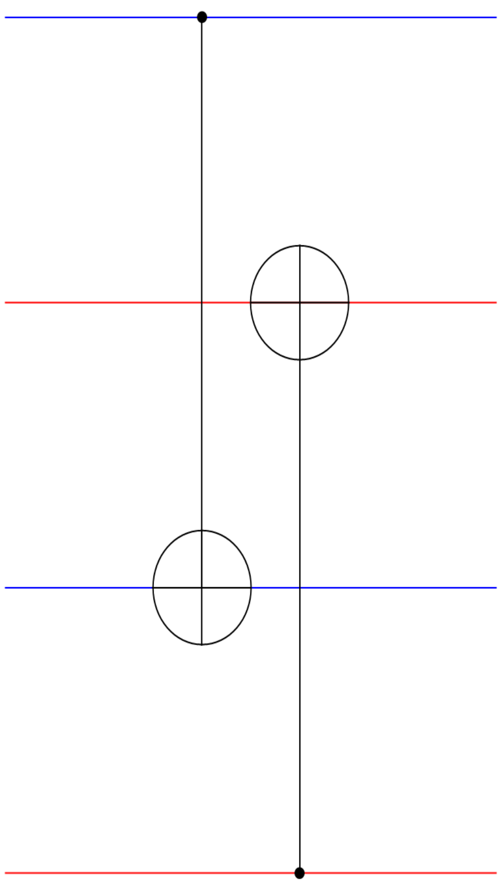
H|1>=|->

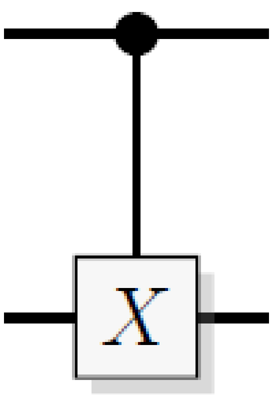
Identities satisfied by one-qubit gates

$$\sigma_x^2 = \sigma_z^2 = H^2 = \mathbb{I}$$



Two-qubit universal gate - **CNOT**

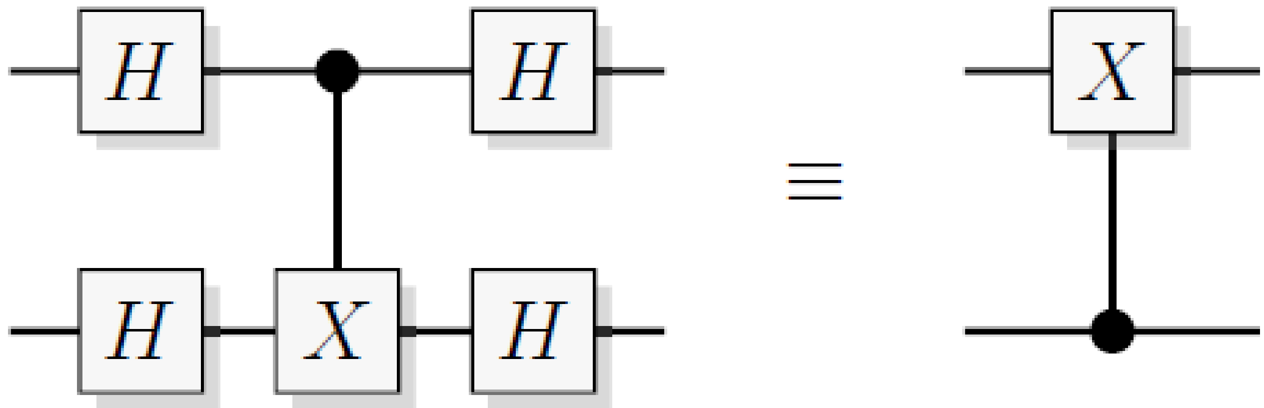


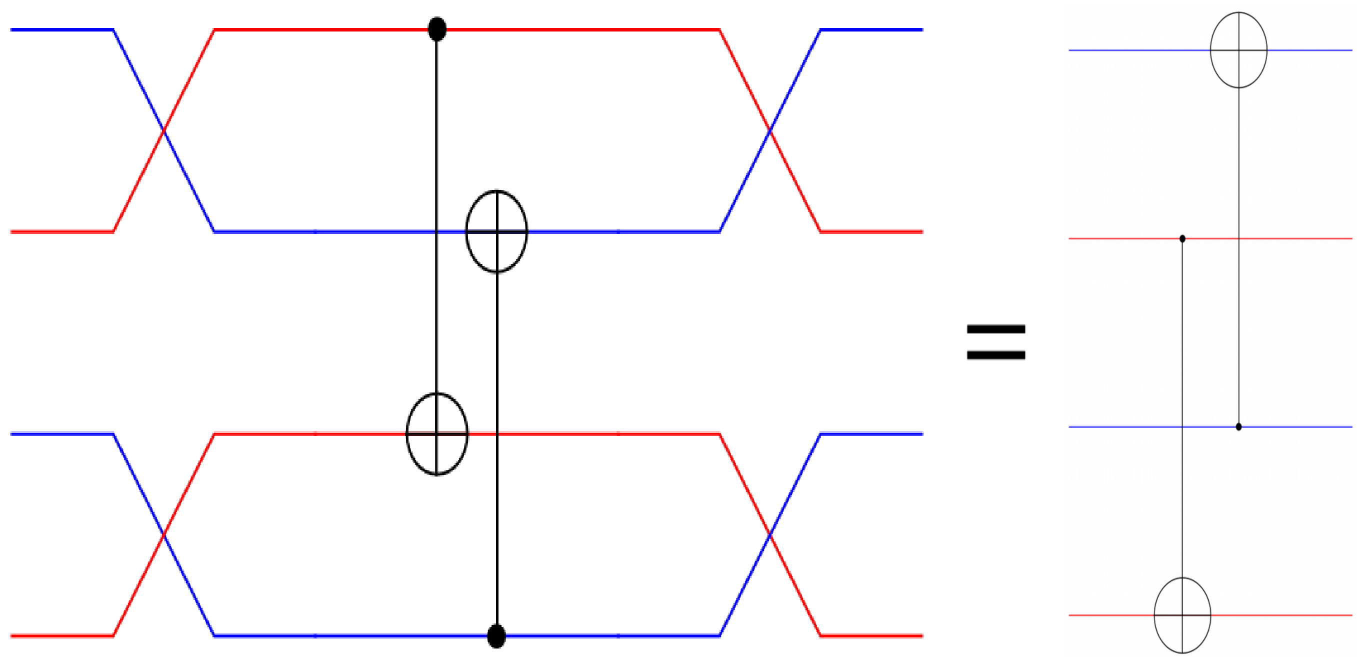


Input		Output	
x	y	x	x+ y
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0

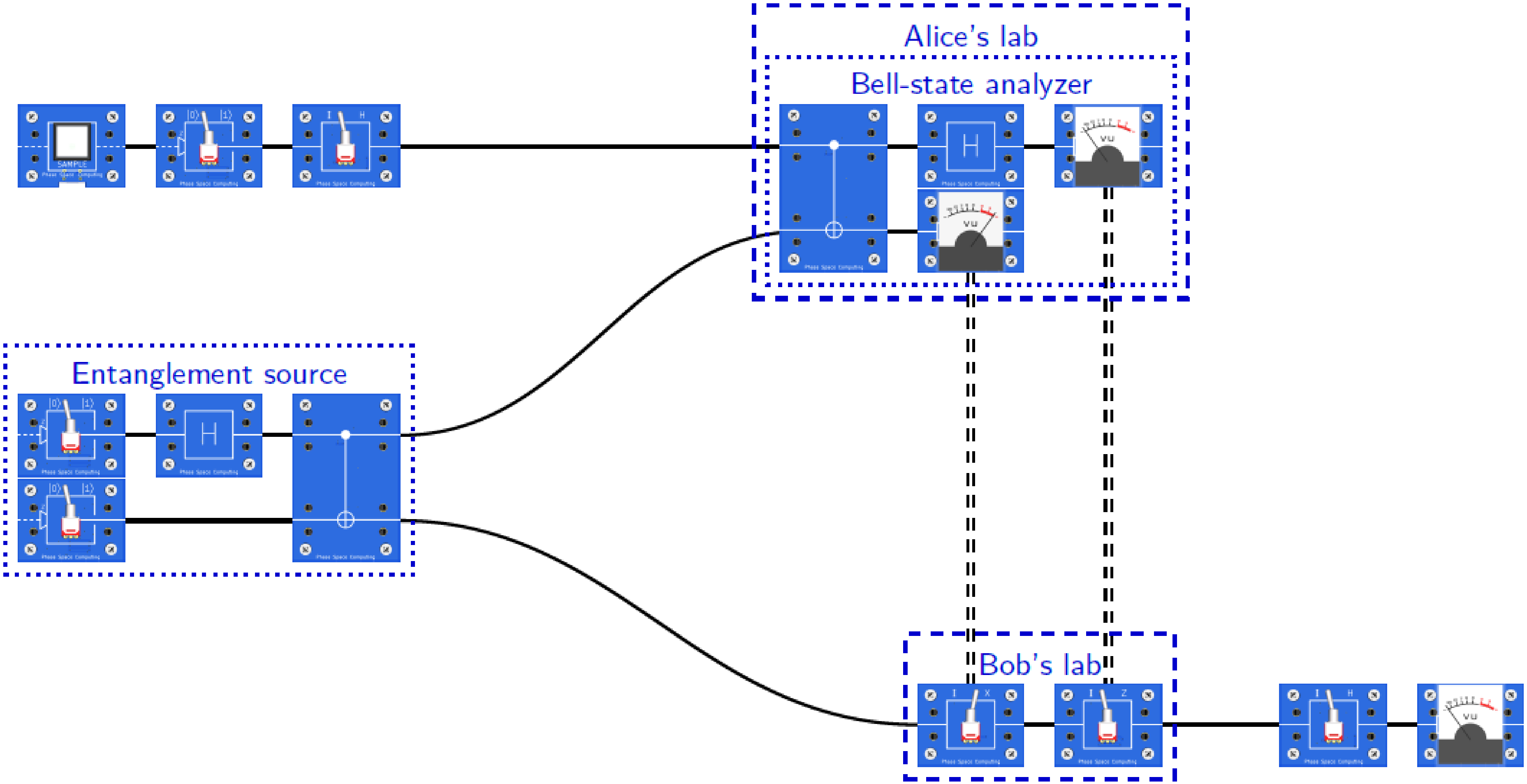
$$CNOT = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

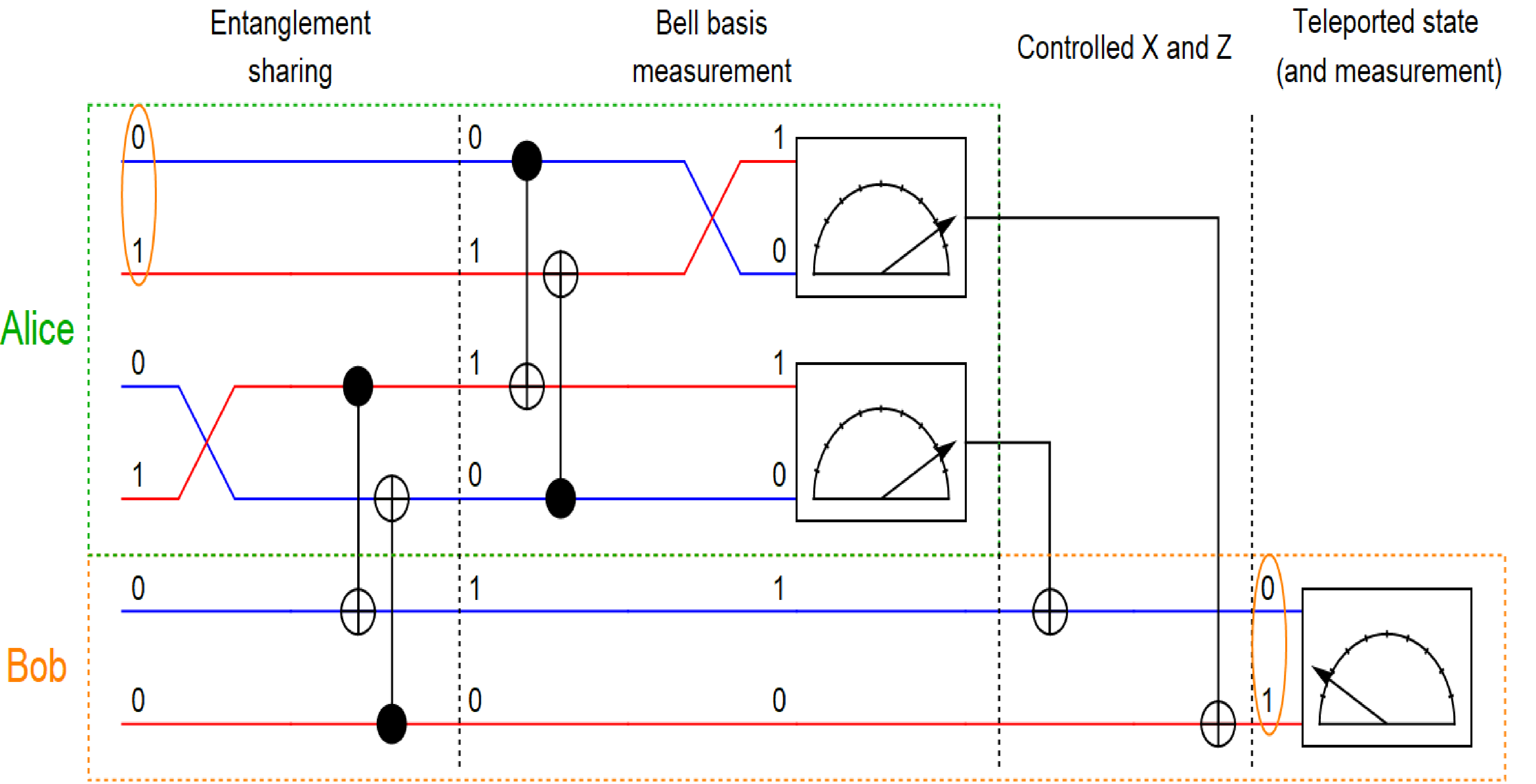
Identity for CNOT and Hadamards



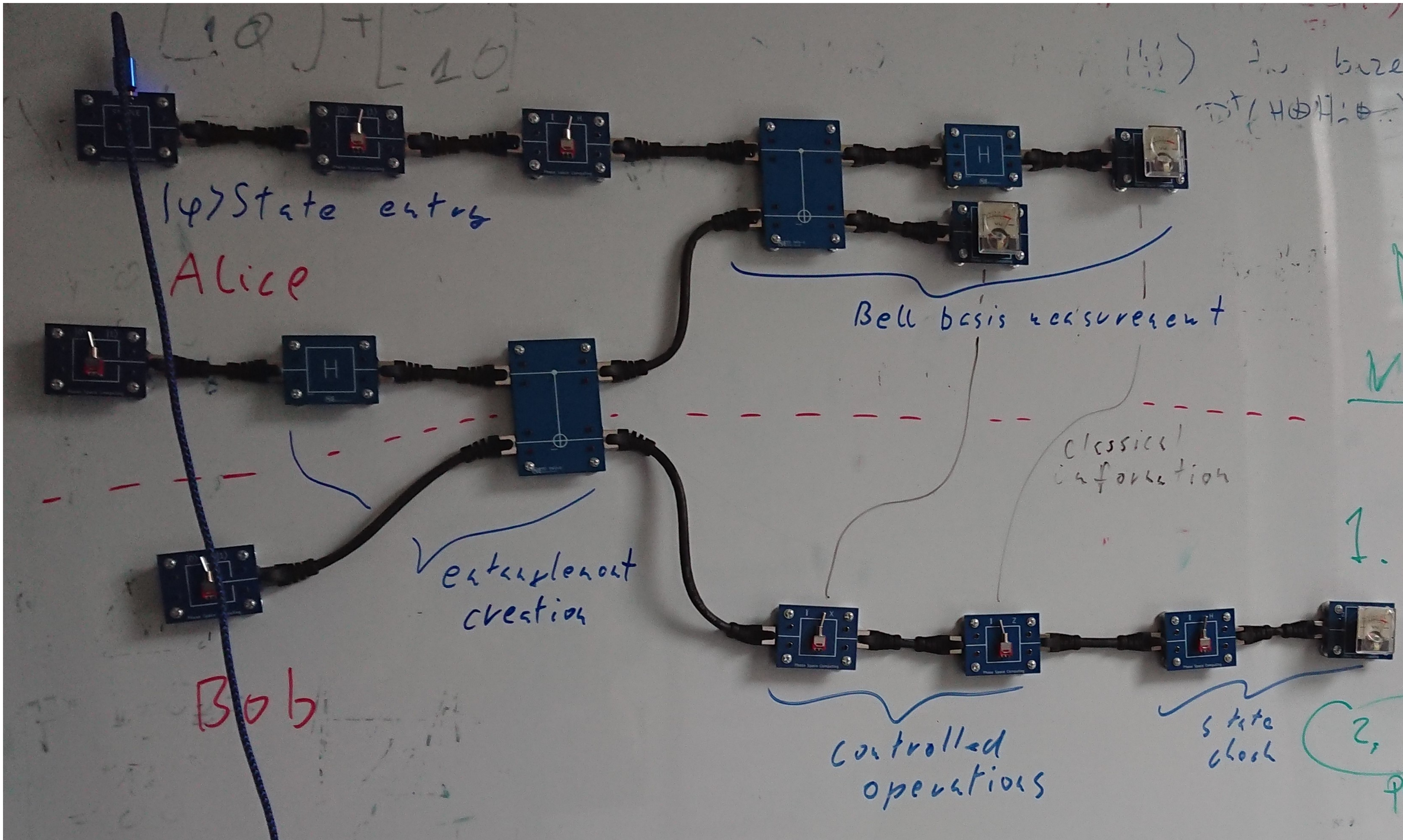


Quantum teleportation: simulation scheme

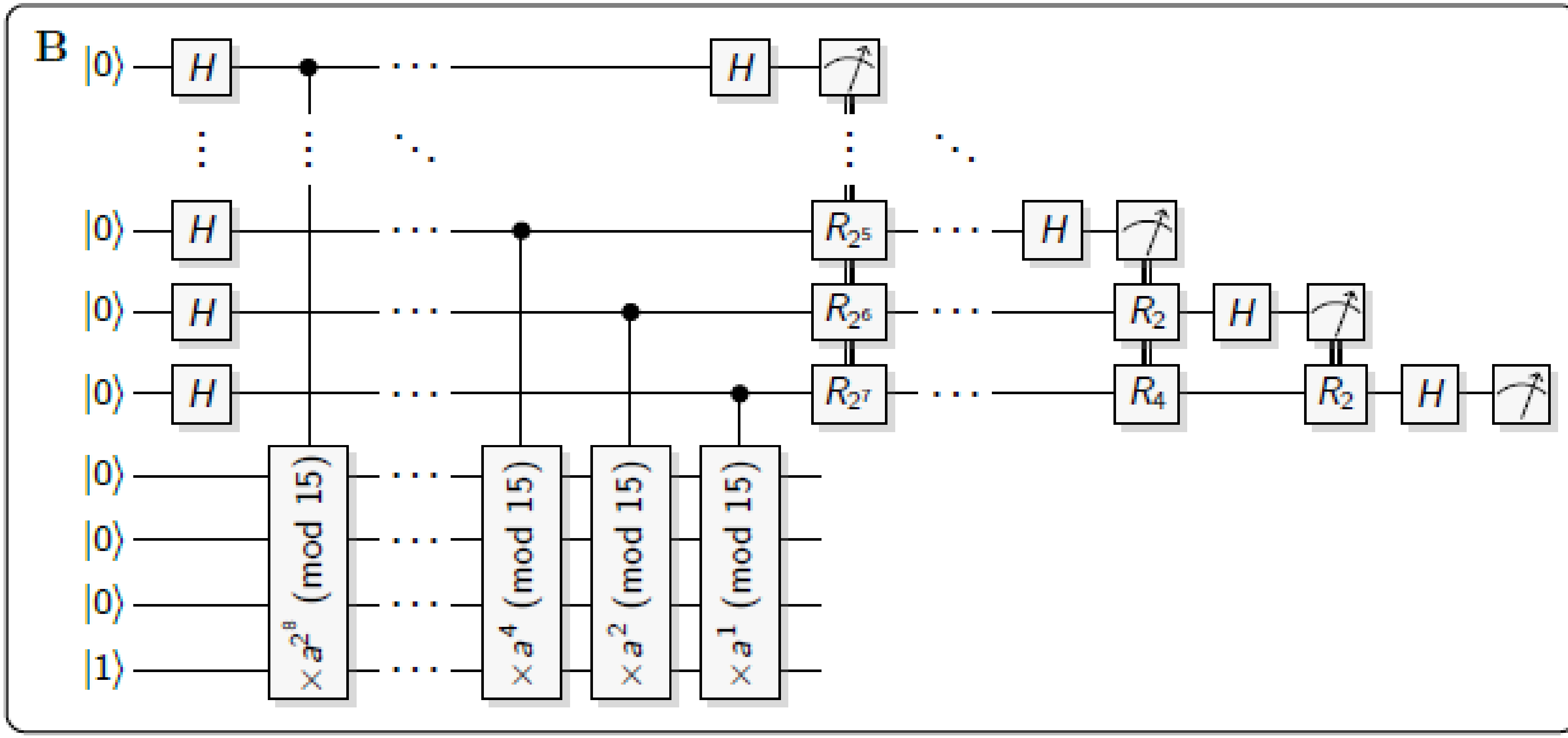



Entanglement sharing
Bell basis measurement
Controlled X and Z
Teleported state (and measurement)
Alice
Bob
0
1
0
1
0
0
0
1
1
0
1
0
1
0
1
0
1
0
1
0
0
1

Quantum teleportation simulated at the board



The scheme of *Shor's Algorithm,*

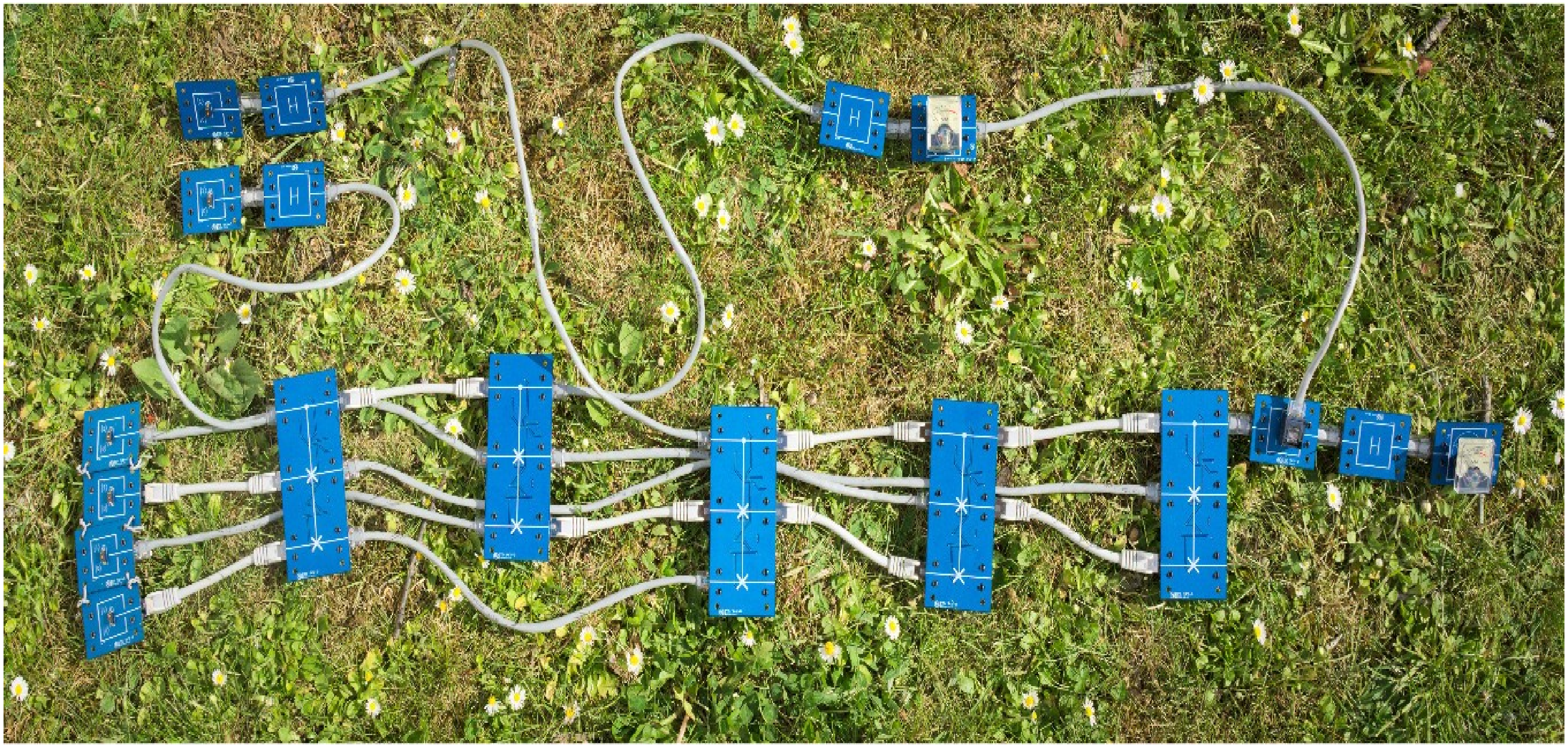


presented here for N=15 can be directly realized with the simulator !

Realization of Shor's Algorithm at room temperature

Niklas Johansson and Jan-Ake Larsson, arXiv: 1706.0321

factorisation of 15=3 x 5 with the simulator



controlled multiplication 2^x mod 15

Concluding Remarks

- *Quantum entanglement* is often presented as the indispensable resource for quantum information processing.
- *Entanglement* is indeed used by **teleportation**, realized in the lab, between cities, islands and ground-to-satellite.
- However, many realizations **(e.g. NMR quantum computing)** in fact do not rely on *entanglement*.
- Quantum information processing can be simulated classically! Quantum simulation logic of Johansson and Larsson allows one to mimic quantum teleportation, and to realize Deutsch - Jozsa and *Shor's algorithms.*
- *Which resources are really indispensable for quantum computing?* (A crucial question still open !)

Key references

- **R. Spekkens**, *In defense of the epistemic view of quantum states: a toy theory,*

  Phys. Rev. A 75, 032110 (2007)
- **N. Johansson and J. Larsson**, *Efficient classical simulation of the Deutsch-Jozsa and Simon's algorithms,*

  Quantum Information Processing 16:233 (2017)
- **N. Johansson and J. Larsson**, *Realization of Shor's Algorithm at Room Temperature,*

  arXiv:1706.03215 [quant-ph]