Stochastic Local Operations with Classical Communication of Absolutely Maximally Entangled States

Adam Burchardt

Joint work with Zahra Raissi

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BIPARTITE SYSTEMS

Bipartite systems

Bell state $\mathcal{H}_2 \otimes \mathcal{H}_2$

$$|\Psi^+\rangle = \frac{1}{\sqrt{2}}\Big(|00\rangle + |11\rangle\Big)$$

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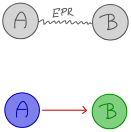

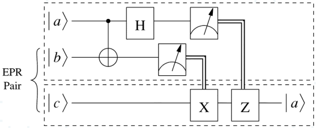

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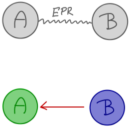

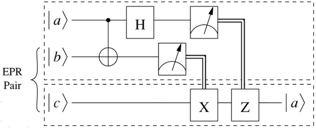

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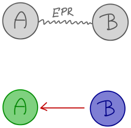

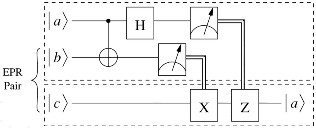

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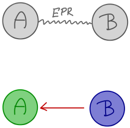

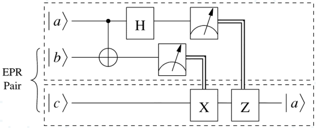

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$$\rho_1 := \mathrm{tr}_2\,|\psi_d^+\rangle\langle\psi_d^+| \sim \mathrm{Id}_d$$

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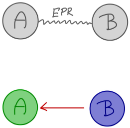

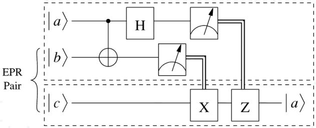

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- Maximal entanglement entropy

  $$E(|\psi_d^+\rangle) = \log d$$

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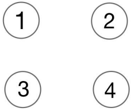

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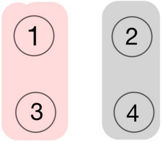

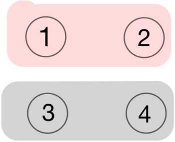

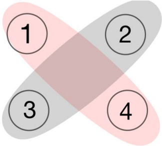

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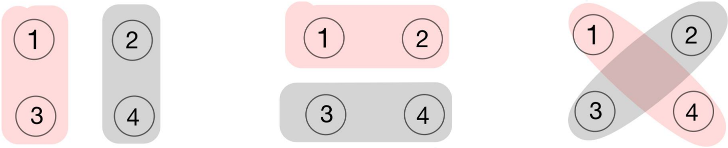

Generalized GHZ state? $\mathcal{H}_2 \otimes \mathcal{H}_2 \otimes \mathcal{H}_2 \otimes \mathcal{H}_2$

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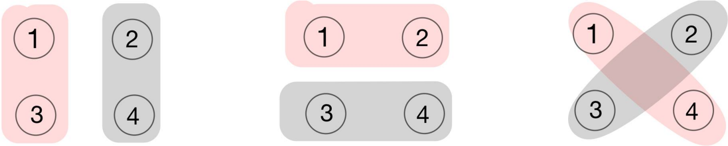

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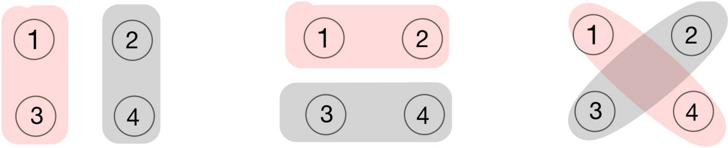

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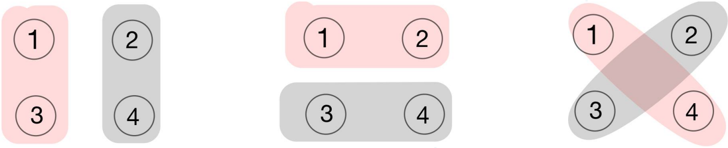

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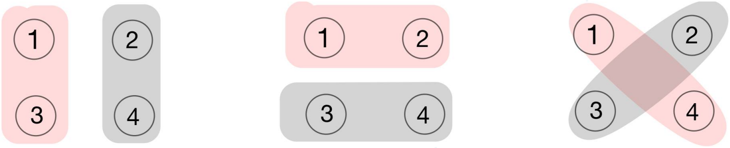

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$$\begin{aligned} \rho_{12}(\psi) &= \rho_{34}(\psi) = \mathrm{Id}_9 \\ \rho_{13}(\psi) &= \rho_{24}(\psi) = \mathrm{Id}_9 \\ \rho_{14}(\psi) &= \rho_{23}(\psi) = \mathrm{Id}_9 \end{aligned}$$

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Pure state $|\psi\rangle \in \mathcal{H}_d^{\otimes N}$. Reduced density matrix

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- 2-uniform state of 4 **qutrits** does exist.

2-uniform state

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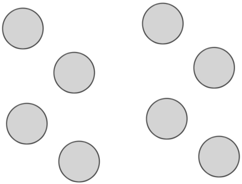

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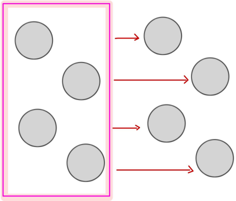

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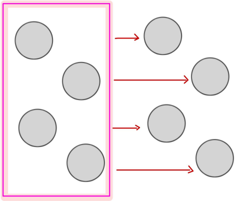

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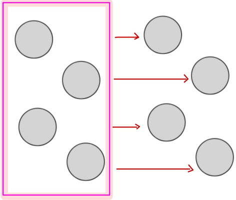

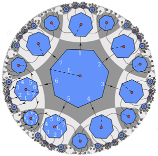

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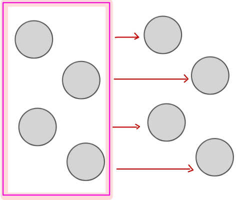

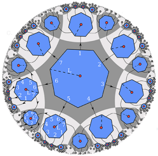

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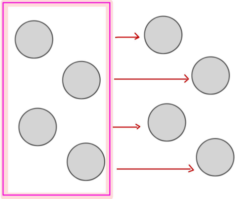

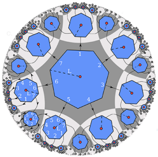

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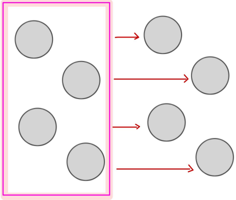

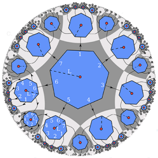

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0	2	2	2	$\|0222\rangle$
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1	1	0	2	$\|1120\rangle$
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IrOA

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1	1	0	2	$\|1120\rangle$
1	2	0	1	$\|1201\rangle$
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- A 2-uniform state
- States with the minimal support

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EQUIVALENCE CLASSES

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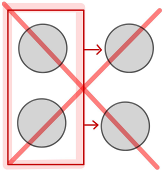

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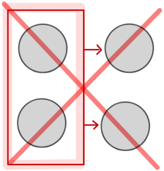

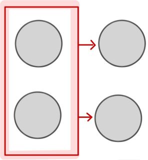

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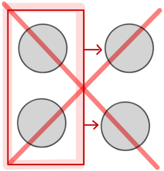

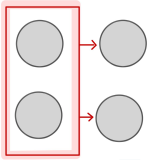

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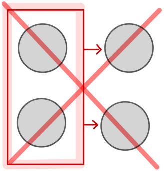

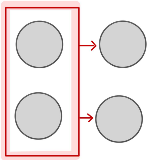

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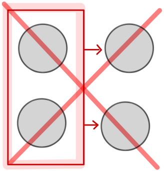

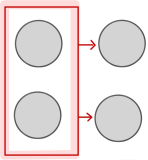

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- Invariant methods are unsatisfactory. 3 qubits: $\mathbb{R}^5$

SLOCC verification of min. supp. states

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For a local dimension $d \geq 5$, the states $|\psi_{5,0}\rangle$ and $|\psi_{5,2}\rangle$ are not LU.

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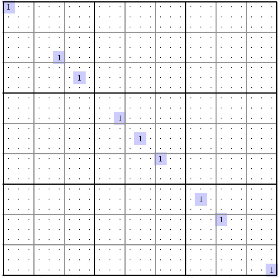

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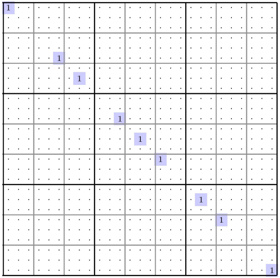

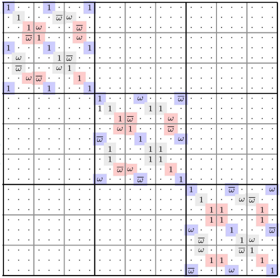

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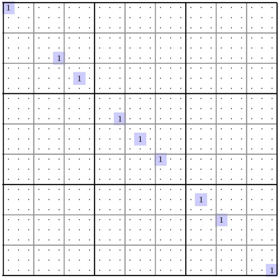

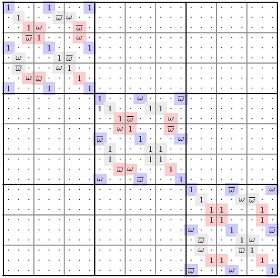

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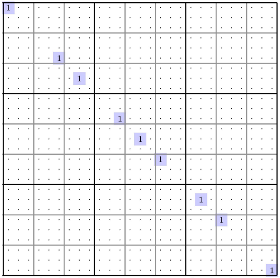

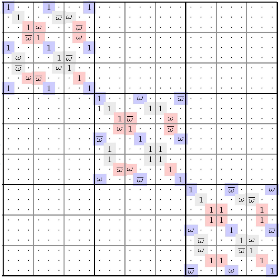

- $\rho_{345}(\psi_{5,0}), \rho_{345}(\psi_{5,2})$
- They are equivalent!
- But there is no extension

Different phases - Example

Different phases - Example

AME(4,3) states

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$$\rho_{12}(\psi) = \rho_{34}(\psi) = \mathrm{Id}_9$$
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AME(5,d)	0	0	1	1	0	1
AME(6,d)	0	0	∞	∞	0	∞
AME(7,d)	0	0	0	0	0	∞

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GRAPH STATES

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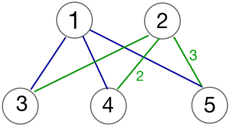

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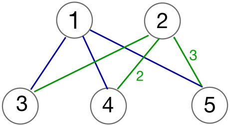

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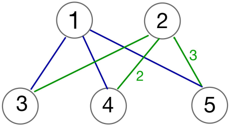

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Depth of a quantum circuit

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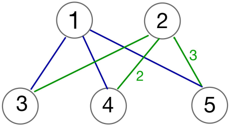

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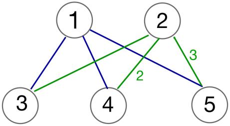

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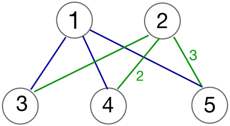
Graph states



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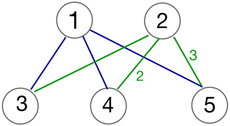
Graph states



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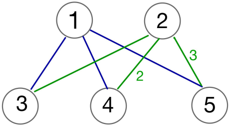

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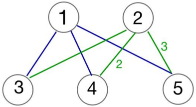

AME(5,5) min. supp.

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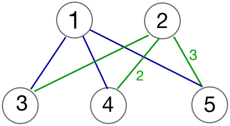

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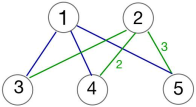
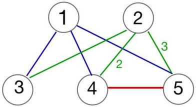
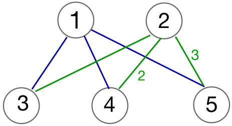


$\longmapsto$



AME(5,5) min. supp. AME(5,5) non min. supp.

GRAPH STATES



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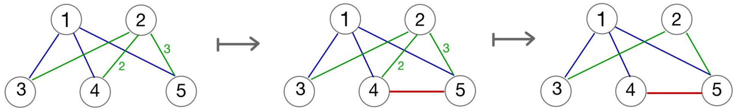

AME(5,5) min. supp. | AME(5,5) non min. supp. | AME(5,2) non min. supp.

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- If there exists one AME(2k,d) state of the minimal support for $k > 2$, there are infinitely many of them. How they can be parametrized?

SUMMARY

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Thank You!