



Fundacja na rzęcz Nauki Polskiej

Mitigation of measurement noise on quantum devices

Filip Maciejewski, Flavio Baccari, Zoltán Zimborás, Michał Oszmaniec



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Outline of presentation

Measurement noise + mitigation

PART I – on small systems

PART II - on big systems

PART I is based on:



Mitigation of readout noise in near-term quantum devices by classical post-processing based on detector tomography

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20 We propose a simple scheme to reduce readout errors in experiments on quantum systems with finite number of measurement outcomes. Our method relies on performing classical postprocessing which is preceded by Quantum De-3 tector Tomography, i.e., the reconstruction of a Positive-Operator Valued Measure (POVM) describing the given quantum measurement device. If the measurement device is affected only by an invertible classical noise, it is possible to correct the outcome statistics of future experiments performed on the same device. To support the practical applicability of this scheme for nearterm quantum devices, we characterize measurements implemented in IBM's and Rigetti's quantum processors. We find that for these devices, based on superconducting transmon qubits, clas-

sical noise is indeed the dominant source of readout errors. Moreover, we analyze the influ-00 ence of the presence of coherent errors and finite statistics on the performance of our error-5 mitigation procedure. Applying our scheme on 00 \bigcirc the IBM's 5-qubit device, we observe a significant improvement of the results of a number of single- and two-qubit tasks including Quantum State Tomography (QST), Quantum Process Tomography (QPT), the implementation of nonprojective measurements, and certain quantum algorithms (Grover's search and the Bernstein-Vazirani algorithm). Finally, we present results showing improvement for the implementation of certain probability distributions in the case of five qubits.



Figure 1: Pictorial representation of our error-mitigation procedure. (i) In the first stage, one performs the tomography of a noisy detector $\mathbf{M}^{\text{noisy}}$ (red semicircle). (ii) In the next stage, when measuring an arbitrary quantum state ρ , one employs a post-processing procedure on the measured statistics through the application of Λ^{-1} , the inverse of a stochastic noise map obtained in the QDT. This gives access to the statistics that would have been obtained in an ideal detector $\mathbf{M}^{\text{ideal}}$ (green semicircle).

of delicate quantum states with unprecedented precision [1]. Due to the advent of quantum cloud services (IBM [2, 3], Rigetti [4], DWave [5]), any researcher has a possibility to perform experiments on actual quantum devices. However, if one really hopes for utilizing such near-term devices for real-life applications such as quantum computation [6], quantum simulations [7] or generating random numbers [8], experimental imperfections must be taken into account. Hence, to properly characterize noise occurring in the devices and to develop error correction and mitigation schemes that may help to fight it have become tasks of fundamental importance [9, 10, 11, 12, 13, 14, 15]. In the present work, we address this problem for the

Quantum 4 257 (2020)



Outline of PART I

Measurement noise + mitigation on small systems

Basic definitions
 Noise model and mitigation.
 Quantum Detector Tomography.
 Some error analysis.

Basic definitions – measurements

• Quantum measurement with n outcomes is a vector of operators:

 $M = (M_1, \dots, M_n)$

s.t.

 $\forall M_i = 1$

Basic definitions - projective measurements

Projective measurements fulfill additional requirement:

 $P = (P_{n}, \dots, P_{n})$

S.t.

 $\forall P_i \gamma Q_i \qquad \mathcal{E} P_i = 1$

Basic definitions - projective measurements

Projective measurements fulfill additional requirement:

 $P = (P_{\gamma}, \dots, P_{m})$

Basic definitions - Born's rule

 If we perform measurement M
 on quantum state ρ, probability of obtaining outcome "i" is given by:

 $p(i|M,g) = T_{r}(g|M,)$

Outline of PART I

Measurement noise + mitigation on small systems

- Basic definitions
 Noise model and mitigation.
 Quantum Detector Tomography.
- 4. Short error analysis.

We want to perform projective measurement:

Our device performs noisy measurement:

NOISE

Our device performs noisy measurement:



Our device performs noisy measurement:

NOISE \uparrow \uparrow

STOCHASTIC MAP

On the level of probability vectors:

On the level of probability vectors:

• On the level of probability vectors:

PROBABILITY K C D ON IDEAL DEVICE

On the level of probability vectors:

QA CLASSICAL. VP NOISE PROBABILIT VECTOR IDEAL DEVICE

• On the level of probability vectors:

(P) CLASSICAL NOISE PROBABILI VECTOR IDEAL DEVICE

NOISE AS POST-PROCESSING

[07]

107 MEASURE

107 MEASURE 10 11

107 MEASURE 10 RELABEL

107 MEASURE RELABEL 9(010) 9,(110

107 MEASURE RELABEL 9,(010)





• On the level of probability vectors:

(P) CLASSICAL NOISE PROBABILI VECTOR IDEAL DEVICE

NOISE AS POST-PROCESSING

Mitigation of classical noise

• On the level of probability vectors:

QP CLASSICAL NOISE PROBABILIT VECTOR 1-7 MITIGATION BY. IDEAL DEVICE POST - PROCESSING

Mitigation of classical noise

This is possible provided we know noise matrix Λ!

Mitigation of classical noise

This is possible provided we know noise matrix A!

How to reconstruct it (and verify noise model)?

Outline of PART I

Measurement noise + mitigation on small systems

- 1. Basic definitions
- 2. Noise model and mitigation.
- 3. Quantum Detector Tomography.
- 4. Some error analysis.

Quantum Detector Tomography (QDT) - basic idea

Put inside a measurement device various quantum states,

Quantum Detector Tomography (QDT) - basic idea

 $A \rightarrow \forall p(i \mid S_1, M)$

 $\overline{\mathcal{A}} \longrightarrow \overline{\mathcal{A}} \longrightarrow \left(\begin{array}{c} \mathbf{i} & \mathbf{i} \\ \mathbf{j} & \mathbf{j} \\ \mathbf{k} & \mathbf{k} \\ \mathbf{k} & \mathbf{k} \end{array} \right)$

 Put inside a measurement device various quantum states, estimate probabilities...
Quantum Detector Tomography (QDT) - basic idea

...and use Born's rule to reconstruct measurement operators:

 $\forall p(\mathbf{n} | \mathbf{S}_{11} | \mathbf{M}) = \int \mathcal{T}(\mathbf{S}_{1} | \mathbf{M}_{1})$

Quantum Detector Tomography (QDT) - basic idea

...and use Born's rule to reconstruct measurement operators:

 $\forall p(\mathbf{n} | \mathbf{s}_{11} \mathbf{M}) = \int_{\mathbf{T}} (\mathbf{s}_{11} \mathbf{M}_{i})$ a Hilbert - Schmidt inner product

How it looks in practice?

Recall: ST OCHASTIC MAP NOISE , AP

 $P_{-} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

 $) = \begin{pmatrix} 0 & 0 \\ 2 & 0 \end{pmatrix}$

 $P_{-} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

 $P = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \xrightarrow{A} M_{1} = \begin{pmatrix} p(0|0) & 0 \\ 0 & p(0|1) \end{pmatrix}$

 $P_{2} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \xrightarrow{A} M_{1} = \begin{pmatrix} p(0|0) & 0 \\ 0 & p(0|1) \end{pmatrix}$

 $D_{2} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \xrightarrow{1} M_{2} = \begin{pmatrix} p(110) & 0 \\ 0 & p(111) \end{pmatrix}$

 Hence if classical noise model is accurate, we expect reconstructed operators to be diagonal...

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- How it looks in practice?

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- How it looks in practice?
- Yesterday evening, the measurement on qubit number 9 on Melbourne device looked like this:

- Hence if classical noise model is accurate, we expect reconstructed operators to be diagonal...
- How it looks in practice?
- Yesterday evening, the measurement on qubit number 9 on Melbourne device looked like this:

 $M_{2} = \begin{pmatrix} 0.943 & 0.001(1-x) \\ 0.001(1+x) \\ 0.001(1+x) \end{pmatrix} = \begin{pmatrix} 0.057 & -0.001(1-x) \\ 0.001(1+x) \\ 0.909 \end{pmatrix}$

- Hence if classical noise model is accurate, we expect reconstructed operators to be diagonal...
- How it looks in practice?

0.001 (1-i

0.001(1+x). 0.101

 Yesterday evening, the measurement on qubit number 9 on Melbourne device looked like this: 5MALL

NOT ZERO

-0.001 (1-

Hence classical noise model is good, but not perfect...

Hence classical noise model is good, but not perfect...
How coherent noise affect error mitigation?

Outline of PART I

Measurement noise + mitigation on small systems

Basic definitions
 Noise model and mitigation.
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 Some error analysis.

Mitigation of classical noise

• On the level of probability vectors:

QP CLASSICAL NOISE PROBABILIT VECTOR 1-7 MITIGATION BY. IDEAL DEVICE POST - PROCESSING

Presented error mitigation method is not perfect because:

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 – noise is not completely classical,

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 – noise is not completely classical,



NOISK MEAUSEREMENT

1 p

NOISY MEAUSEREMENT

NOISY CLASSIC MEAUSEREMENT NOISE

CLASSICAL

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MEAUSEREMENT NOISE

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4

DEVIATION

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E = D_I(<u>A</u> proving pricleM

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CORRECTED

Monorly =

 $\leq || \Lambda^{-1} ||_{1+1} D_{0}(M, \Lambda \vec{P})$

7 iclem

IDEAL PD

+ / /

DEVIATION

CORRECTED

PD

A proisy =

 $\begin{cases} \leq || \Lambda^{-1} ||_{1 \to 1} D_{0}(\Lambda, \Lambda \vec{P}) \end{cases}$

MEAUSEREMENT NOISE

CLASSICAL

A P

COHERENI NOISE

Presented error mitigation method is not perfect because:
 – noise is not completely classical,

Presented error mitigation method is not perfect because:

- noise is not completely classical,
- estimated statistics are not probabilities.

Illustration of statistical errors

NOISY PD

IDEAL PD
IDEAL PD



NOISY PD

IDEAL PD

I PEAL PD

ESTIMATED PD

NOISY PD

I PEAL PD

ESTIMATED PD E² NOISY PD



ESTIMATED PD e² NO15.1/ PD CORRECTED RUASI PD

I DEAL PD

ESTIMATED PD E E NOIS! PD CORRECTED S QUASI PD

IDEAL

PD

CLOSEST PD

CORRECTED ESTIMATED PD · QUASI PD \propto NOISY PD CLOSEST IDEAL

PD

ESTIMATED PD e² NOISV PD

CORRECTED QUASI PD X CLOSEST DEAL



Effects of coherent errors

 $\begin{cases} \leq || \Lambda^{-1} ||_{1 \to 1} D_{0}(\Lambda, \Lambda \vec{P}) \end{cases}$

MEAUSEREMENT NOISE

CLASSICAL

A P

COHERENI NOISE

Effects of coherent and statistical errors

 $\begin{cases} \leq || \Lambda^{-1} ||_{1 \to 1} \left(D_{op}(M, \Lambda \vec{P}) + \xi \right) + \zeta \end{cases}$

COHERENT ERPORS

STATISTICAL ERRORS

What can go wrong?

Presented error mitigation method is not perfect because:

- noise is not completely classical,
- estimated statistics are not probabilities.

What can go wrong?

Presented error mitigation method is not perfect because:

- noise is not completely classical,
- estimated statistics are not probabilities.

So does this actually work?

- - CORR

INFIDELITY

WITH MITIGATION

Single-qubit quantum state tomography.
Figure of merit: infidelity.





Two-qubit quantum state tomography.
Figure of merit: infidelity.







Two-qubit quantum algorithms: Grover's and Bernstein–Vazirani.
Figure of merit: probability of success.



CORRELATED

UNCORRELATED NOISE MODEL

Algorithm Standard Corr $(1q \otimes 1q)$ Corr (2q)

Grover's 0.58 ± 0.01 0.70 ± 0.02 0.79 ± 0.02 BV 0.55 ± 0.02 0.63 ± 0.02 0.61 ± 0.02

- Implementation of five-qubit probability distributions.
- Figure of merit: Total-Variation Distance from perfect distribution.





I UNIFORM

NOT

UNCORRELATED NOISE MODEL

Name	Standard	Corrected	α
Uniform	0.110 ± 0.006	0.100 ± 0.007	0
NOT	0.66 ± 0.02	0 ± 0	0.36 ± 0.09
Mixed	0.196 ± 0.006	0.031 ± 0.008	0.019 ± 0.005



IUNIFORM", NOT"



CORRECTED ESTIMATED PD QUASI PD X NOISY PD CLOSEST DEAL $\times + \parallel \Lambda^{-1} \parallel_{1}$

UNCORRELATED NOISE MODEL

Name	Standard	Corrected	α
Uniform	0.110 ± 0.006	0.100 ± 0.007	0
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UNCORRELATED NOISE MODEL

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Name	Corrected	lpha
Uniform	0.03 ± 0.02	0
NOT	0.004 ± 0.023	0.04 ± 0.04
Mixed	0.022 ± 0.007	0.023 ± 0.007

CORRELATED (1 PAIR)



IUNIFORM" NOT"

Outline of presentation

Measurement noise + mitigation

PART I - on small systems

PART II – on big systems

Outline of PART I

Measurement noise + mitigation on big systems

Problems with noise description + solution.
 Problems with noise mitigation + "solution".

Mitigation of classical noise

• On the level of probability vectors:

QP CLASSICAL NOISE PROBABILIT VECTOR 1-7 MITIGATION BY. IDEAL DEVICE POST-PROCESSING

 $M \in \mathbb{R}^{2^{m} \times 2^{m}}$

Problem: Requires reconstruction of generic noise matrix.

Problem: Requires reconstruction of generic noise matrix.

A E R²×2 N E R²×2 SI EXPONENTIAL

1, DESCRIPTION

• Problem: Requires reconstruction of generic noise matrix. $A \in \mathbb{R}^{2 \times 2}$ $E \times PONENTIAL$

• Problem: Requires reconstruction of generic noise matrix. $A \in \mathbb{R}^{2 \times 2}$ $E \times PONENTIAL$

2. NUMBER OF CIRCUITS

1. DESCRIPTION

Quantum Detector Tomography complexity

Number of states we need to prepare is in general equal to 4^N.
Quantum Detector Tomography complexity

Number of states we need to prepare is in general equal to 4^N.
 However...

 ...if we checked that noise is classical via single-qubit Quantum Detector Tomography...

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OHERE

NOIST CLASSICAL MEAUSEREMENT NOISE

 ...if we checked that noise is classical via single-qubit Quantum Detector Tomography...

NOIST CLASSICAL MEAUSEREMENT NOISE

 ... we can restrict ourselves to reconstruction of only diagonal elements of noisy measurement operatos...

OHERE

1q classical noise



1q classical noise



2q noise-matrix characterization via DDT

 $A_{x_1x_2}|_{00}$ 5/ 0 $\Lambda_{X_1X_2}|01$ 1 × $A_{X_1X_2} | 10$ 1 $AX_X_2 | 11$

 ...if we checked that noise is classical via single-qubit Quantum Detector Tomography...

 ... we can restrict ourselves to reconstruction of only diagonal elements of noisy measurement operatos...

 ...if we checked that noise is classical via single-qubit Quantum Detector Tomography...

 ... we can restrict ourselves to reconstruction of only diagonal elements of noisy measurement operatos...

• ... which anyway gives 2^N circuits using DDT.

• Problem: Requires reconstruction of generic noise matrix. $A \in \mathbb{R}^{2 \times 2}$ $E \times PONENTIAL$

2. NUMBER OF CIRCUITS

1. DESCRIPTION

• Problem: Requires reconstruction of generic noise matrix. $A \in \mathbb{R}^{2 \times 2}$ $E \times PONENTIAL$

2. NUMBER OF CIRCUITS

1. DESCRIPTION

3. SAMPLING COMPLEXITY

 $A \in R^{2} \times 2$

Problem: Requires reconstruction of generic noise matrix.

SEXPONENTIAL



Problem: Requires reconstruction of generic noise matrix.

NER^{2×2} Exponential

Solution:

Real noise is unlikely to be generic – we can make use of locality of correlations

Think about the simplest model – uncorrelated noise:

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 $\Lambda = \Lambda^{q_1} \wedge^{\gamma_2} \otimes \Lambda^{q_N}$

Think about the simplest model – uncorrelated noise:

 $\Lambda = \Lambda^{q_1} \wedge^{v_2} \otimes \Lambda^{q_N}$

It is efficient in terms of:

Think about the simplest model – uncorrelated noise:

 $\int - \int \sqrt{2} \otimes \int \sqrt{2}$

It is efficient in terms of:

1, DESCRIPTION

Think about the simplest model – uncorrelated noise:

 $\int - \int \mathcal{Q} \wedge \mathcal{V}^2 \otimes \mathcal{Q} \wedge \mathcal{Q} \wedge$

It is efficient in terms of:

1, DESCRIPTION 2. NUMBER OF CIRCUITS

Think about the simplest model – uncorrelated noise:

 $\int - \int \sqrt{2} \sqrt{2} \sqrt{2} \sqrt{2}$

It is efficient in terms of:

1. DESCRIPTION

2. NUMBER 3. SAMPLING OF CIRCUITS COMPLEXITY

(too) Simple solution

Think about the simplest model – uncorrelated noise:

It is efficient in terms of:

1. DESCRIPTION 2. NUMBER 3. SAMPLING OF CIRCUITS COMPLEXITY

• Unfortunately, it is not very accurate¹!

¹Quantum 4, 257, FBM, Z. Zimborás, M. Oszmaniec (2020).

Model correlations locally, but do not be too naive.

Multi-qubit noise model





 Strongly correlated qubits are grouped into clusters. Each cluster has corresponding noise matrix.

 $C_A = \{Q_3, Q_4\}$

 Strongly correlated qubits are grouped into clusters. Each cluster has corresponding noise matrix.

 The noise matrix in each cluster depends on the state of the neighbors just before measurement.

 $C_A = \{Q_3, Q_4\}$

 Strongly correlated qubits are grouped into clusters. Each cluster has corresponding noise matrix.

 The noise matrix in each cluster depends on the state of the neighbors just before measurement.

 Characterization of clusters and neighbors can be cheap (if they are not too big).



 $\mathcal{C}_{A} = \{Q_{3}, Q_{4}\}$

 $-\chi_{1}\chi_{2}\chi_{3}\chi_{4}|Y_{1}\chi_{2}\chi_{3}|_{4}=$



Global noise with local clusters and neighbors:

Global noise with local clusters and neighbors:

$\Lambda_{X_1...X_N|Y_1...Y_N} = \prod_{C_{\gamma}} \Lambda_{\mathbf{X}_{C_{\chi}}|\mathbf{Y}_{C_{\chi}}}^{\mathbf{Y}_{\mathcal{N}(C_{\chi})}}$

Global noise with local clusters and neighbors:

 $C_{\mathbf{v}}$



It is efficient in terms of:

Global noise with local clusters and neighbors:

 $C_{\mathbf{v}}$



It is efficient in terms of:

1. DESCRIPTION

Global noise with local clusters and neighbors:

 $C_{\mathbf{v}}$

$\Lambda_{X_1...X_N|Y_1...Y_N} = \prod \Lambda_{\mathbf{X}_{C_{\mathbf{Y}}}|\mathbf{Y}_{C_{\mathbf{Y}}}}^{\mathbf{Y}_{\mathcal{N}(C_{\mathbf{X}})}}$

It is efficient in terms of:

1. DESCRIPTION 2. NUMBER OF CIRCUITS

Global noise with local clusters and neighbors:

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It is efficient in terms of:

1. DESCRIPTION 2. NUMBER 3. SAMPLING OF CIRCUITS COMPLEXITY

Multi-qubit noise characterization

- Method of efficiently characterizing readout noise:
 - Perform parallel noise characterization using generalized
 Quantum Overlapping Tomography¹ Diagonal Detector Overlapping Tomography.

¹Phys. Rev. Lett. 124, 100401, J. Cotler, and F. Wilczek (2020)

Diagonal Detector Overlapping Tomography – main idea

Separate DDTs on pairs of qubits require ~N² circuits...
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 $A_{X_1X_2}|_{00}$ 511_ 1 $\Lambda_{X_1X_2}|01$ X 1 1 Ax, Xz 10 × A_{X,X_2}

Separate DDTs on pairs of qubits require ~N² circuits...



Separate DDTs on pairs of qubits require ~N² circuits...
... but if you make it paralel...



- Separate DDTs on pairs of qubits require ~N² circuits...
- ... but if you make it paralel... DDOT uses ~log(N) exp(K) circuits.



Relevant example: for 15 qubits, reconstruction of noise matrices on all fives of qubits requires 2⁵ (¹⁵/₅) ~ 100,000 circuits.

- Relevant example: for 15 qubits, reconstruction of noise matrices on all fives of qubits requires 2⁵ (¹⁵/₅) ~ 100,000 circuits.
- Overlapping tomography allows to do so using ~350 circuits.

- Method of efficiently characterizing readout noise:
 - Perform parallel noise characterization using generalized
 Quantum Overlapping Tomography¹ Diagonal Detector Overlapping Tomography.

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 - If needed, refine a noise model with standard, local DDTs or restricted DDOTs.

- Method of efficiently characterizing readout noise:
 - Perform parallel noise characterization using generalized
 Quantum Overlapping Tomography¹ Diagonal Detector Overlapping Tomography.
 - From results of DDOT get structure of clusters and neighborhoods.
 - If needed, refine a noise model with standard, local DDTs or restricted DDOTs.
 - Construct a noise model.

Experimental results - IBM, 15 qubits



Experimental results - IBM, 15 qubits (clusters)



Experimental results - IBM, 15 qubits (clusters and neighbours)



(~solved) Problems with noise characterization

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- Problem: Requires reconstruction of generic noise matrix.
- Solution: Real noise is unlikely to be generic - we can exploit locality of correlations.

(~solved) Problems with noise characterization

- Problem: Requires reconstruction of generic noise matrix.
- Solution:

Real noise is unlikely to be generic - we can exploit locality of correlations.

See other works on this:

- arXiv:2001.09980
 M.R. Geller, M. Sun (Jan 2020)
- arXiv:2001.09980
 K. E. Hamilton, T. Kharazi, T. Morris, A. J. McCaskey, R. S. Bennink, R.C. Pooser (Jun 2020)
- arXiv:2006.14044

S. Bravyi, S. Sheldon, A. Kandala, D.C. Mckay, J. M. Gambetta (Jun 2020)

Problems with noise-mitigation

Problems with noise-mitigation

 Problem: Requires post-processing of exponentially big noise matrix and exponentially big probability vectors.

Mitigation of classical noise

• On the level of probability vectors:

QP CLASSICAL NOISE PROBABILIT VECTOR 1-7 MITIGATION BY. IDEAL DEVICE POST-PROCESSING

Problems with noise-mitigation

 Problem: Requires post-processing of exponentially big noise matrix and exponentially big probability vectors.

Problems with noise-mitigation

Problem:

Requires post-processing of exponentially big noise matrix and exponentially big probability vectors.

• "Solution" :

Focus on problems which are feasible.

Feasible problems

Feasible problems

 Those which require estimation of marginal probability distributions (for example - estimation of energy of local Hamiltonians).

Feasible problems

 Those which require estimation of marginal probability distributions (for example - estimation of energy of local Hamiltonians).

 $H = \sum_{K} H_{K} = \sum_{i} \sigma_{i} \sigma_{i+1} h_{i}$

Take global noise with local clusters and neighbors:

$\Lambda_{X_1...X_N|Y_1...Y_N} = \prod_{C} \Lambda_{\mathbf{X}_{C_{\chi}}|\mathbf{Y}_{C_{\chi}}}^{\mathbf{Y}_{\mathcal{N}(C_{\chi})}}$



Check how it affects marginal distributions:

 $C_{\mathcal{K}}$

 $= \sum_{\mathbf{Y}_{\mathcal{N}\mathcal{K}}} p\left(\mathbf{Y}_{\mathcal{N}\mathcal{K}} | \mathbf{Y}_{C\mathcal{K}}\right) \prod_{C_{\chi} = C_{1}}^{C_{\mathcal{K}}} \Lambda_{\mathbf{X}_{C_{\chi}}|\mathbf{Y}_{C_{\chi}}}^{\mathbf{Y}_{\mathcal{N}C_{\chi}}}$

Check how it affects marginal distributions:

 $C_{\mathcal{K}}$ $|\mathbf{X}_{\mathbf{C}^{\mathcal{K}}}\rangle = \sum p\left(\mathbf{Y}_{\mathcal{N}^{\mathcal{K}}} | \mathbf{Y}_{C^{\mathcal{K}}}\right) \prod \Lambda_{\mathbf{X}_{C_{\chi}}|\mathbf{Y}_{C_{\chi}}}^{\mathcal{N}_{C_{\chi}}}$ $\mathbf{Y}_{\mathcal{N}}\mathcal{K}$ $C_{\gamma} = C_1$ NOISE MATRIX ON MARGINAL

Check how it affects marginal distributions:

NOISE

MATRIX

ON MARGINAL

 $|\mathbf{X}_{\mathbf{C}^{\mathcal{K}}}| = \sum_{\mathbf{Y}_{\mathcal{N}^{\mathcal{K}}}} p\left(\mathbf{Y}_{\mathcal{N}^{\mathcal{K}}} | \mathbf{Y}_{C^{\mathcal{K}}}\right)$

 $\begin{pmatrix} \mathbf{Y}_{\mathcal{N}_{C_{\chi}}} \\ \boldsymbol{\Lambda}_{\mathbf{X}_{C_{\chi}}}^{\mathbf{Y}_{C_{\chi}}} \\ \mathbf{Y}_{C_{\chi}} \end{pmatrix}$

SMALL MAIRICES

WE GOT FROM

CHARACTERIZATION

Check how it affects marginal distributions:

 $\mathbf{Y}_{\mathcal{N}\mathcal{K}}$

 $\tilde{\Lambda}_{\mathbf{X}_{\mathbf{C}^{\mathcal{K}}}|\mathbf{X}_{\mathbf{C}^{\mathcal{K}}}}^{\mathcal{K}} = \sum \left(p\left(\mathbf{Y}_{\mathcal{N}^{\mathcal{K}}}|\mathbf{Y}_{C^{\mathcal{K}}}\right) \right)$

NOIGE MATRIX ON MARGINAL PERFECT CONDITIONAL DISTRIBUTION

SMALL MATRICES WE GOT FROM CHARACTERIZATION

 $C_{\chi} = C_1$

 $\begin{array}{c} \mathbf{Y}_{\mathcal{N}_{C_{\chi}}} \\ \boldsymbol{\Lambda}_{\mathbf{X}_{C_{\chi}}}^{\mathbf{Y}_{C_{\chi}}} | \mathbf{Y}_{C_{\chi}} \end{array}$

Check how it affects marginal distributions:

 $\tilde{\Lambda}_{\mathbf{X}_{\mathbf{C}^{\mathcal{K}}}|\mathbf{X}_{\mathbf{C}^{\mathcal{K}}}}^{\mathcal{K}} = \sum p\left(\mathbf{Y}_{\mathcal{N}^{\mathcal{K}}}|\mathbf{Y}_{C^{\mathcal{K}}}\right)$

NOIGE MATRIX ON MARGINAL YNK PERFECT ONDITIONAL DISTRIBUTION (UNKNOWN)

SMALL MATRICES WE GOT FROM CHARACTERIZATION

 $C_{\chi} = C_1$

 $\begin{array}{c} \mathbf{Y}_{\mathcal{N}_{C_{\chi}}} \\ \boldsymbol{\Lambda}_{\mathbf{X}_{C_{\chi}}}^{\mathbf{Y}_{C_{\chi}}} | \mathbf{Y}_{C_{\chi}} \end{array}$

Check how it affects marginal distributions:

YNK

 $\Lambda_{\mathbf{X}_{\mathbf{C}^{\mathcal{K}}}|\mathbf{X}_{\mathbf{C}^{\mathcal{K}}}}^{\mathcal{K}} = \sum \left(p\left(\mathbf{Y}_{\mathcal{N}^{\mathcal{K}}}|\mathbf{Y}_{C^{\mathcal{K}}}\right) \right) \prod_{k=1}^{\mathcal{K}} \right)$

PERFECT

CONDITIONAL

DISTRIBUTION

(UNKNOWN

 $C_{\mathcal{K}}$

 $C_{\gamma} = C_1$

 $\Lambda_{\mathbf{X}_{C_{\mathbf{Y}}}|\mathbf{Y}_{C_{\mathbf{X}}}}^{\mathbf{Y}_{\mathcal{N}_{C_{\mathbf{X}}}}}$

Check how it affects marginal distributions:

=

 $\mathbf{Y}_{\mathcal{N}}\mathcal{K}$

PFRFECT

CONDITIONAL

DISTRIBUTION

CUNKNOWN

 $p(\mathbf{Y}_{\mathcal{N}^{\mathcal{K}}} | \mathbf{Y}_{C^{\mathcal{K}}})$

 $\tilde{\Lambda}_{\mathbf{X}_{\mathbf{C}\mathcal{K}}}^{\mathcal{K}} | \mathbf{X}_{\mathbf{C}\mathcal{K}} |$

 $C_{\mathcal{K}}$

 $C_{\gamma} = C_1$

 $\begin{array}{c} \mathbf{Y}_{\mathcal{N}_{C_{\chi}}}\\ \boldsymbol{\Lambda}_{\mathbf{X}_{C_{\chi}}}^{\mathbf{Y}_{\mathcal{N}_{\chi}}} \\ \end{array}$

Check how it affects marginal distributions:

 $\mathbf{Y}_{\mathcal{N}}\mathcal{K}$

PERFECT

CONDITIONAL

DISTRIBUTION

CUNKNOWN

 $\tilde{\Lambda}_{\mathbf{X}_{\mathbf{C}\mathcal{K}}}^{\mathcal{K}} | \mathbf{X}_{\mathbf{C}\mathcal{K}}|^{\Xi}$

 $p(\mathbf{Y}_{\mathcal{N}^{\mathcal{K}}}|\mathbf{Y}_{C^{\mathcal{K}}})$

 $C_{\mathcal{K}}$

 $C_{\gamma} = C_1$

 $\begin{array}{c} \mathbf{Y}_{\mathcal{N}_{C_{\chi}}} \\ \boldsymbol{\Lambda}_{\mathbf{X}_{C_{\chi}}}^{\mathbf{Y}_{C_{\chi}}} | \mathbf{Y}_{C_{\chi}} \end{array}$

PUT HERE

UNIFORM

In this way you obtain average noise matrix / on marginal.

In this way you obtain average noise matrix for marginal.
And you can use its inverse for as a correction for that marginal.
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- This will not perfectly reverse the noise, but it should bring us closer to the noiseless distribution (up to some error which we quantify).

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DISTANCE FROM IDEAL DISTRIBUTION

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DISTANCE FROM IDEAL DISTRIBUTION

 $\int_{L} \leq \| \Lambda \|_{\Lambda + 1} \left(\int_{L} \int_{$

 $\int_{E} \leq \| \int_{A} \| \int_{A} \| \int_{A} \left(\sum_{i=1}^{n} \left($

DISTANCE

FROM IDEAL DISTRIBUTION

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 $\int_{k} \leq \| \Lambda \|_{\Lambda + 1} \left(\frac{\tilde{\epsilon}}{\epsilon} + \right)$

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DISTANCE FROM IDEAL DISTRIBUTION STATISTICAL ERRORS

In this way you obtain average noise matrix A on marginal.

 $\int_{K} \leq \| \tilde{A}^{\dagger} \|_{A+q} \left(\tilde{E} + \max \| \tilde{A} - \tilde{A}^{\prime} \|_{A+q} \right)$

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DISTANCE FROM IDEAL DISTRIBUTION STATISTICAL ERRORS

In this way you obtain average noise matrix / on marginal.

 $\mathcal{E}_{k} \leq \|[\Lambda]\|_{\Lambda \neq j} \left(\tilde{\mathcal{E}} + \max \|[\Lambda - \Lambda^{k}]\|_{\eta} \right)$

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DISTANCE FROM IDEAL DISTRIBUTION STATISTICAL ERRORS

LAPPROXIMATION ERROR

Proposition for benchmark of the noise model and error-mitigation:

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 - Prepare classical state which is a ground state of some local Hamiltonian.

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 - Prepare classical state which is a ground state of some local Hamiltonian.
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- We did it for 600 Hamiltonians encoding random MAX 2SAT (clause density 4) instances and fully connected random 2-local Hamiltonians on IBM's 15Q Melbourne device.

Experimental results - IBM, 15 qubits (MAX 2SAT)



Experimental results - IBM, 15 qubits (random 2-local)



Summary of presentation

Modeling and mitigation of noise in quantum measurements.

- Modeling and mitigation of noise in quantum measurements.
- Reconstruction of noise using Quantum Detector Tomography.

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QUANTUM 4257

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Additional slides

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SOON

- Successful noise-mitigation in experiments on 15 qubits.
- Effects of approximate mitigation and statistical errors.
- Effects of realistic readout noise on Quantum Approximate Optimization Algorithm.
- Statistical analysis of estimation of energy of local Hamiltonians.

Check out our GitHub repository: github.com/fbm2718/QREM