

A robust approach to QUBO on the chimera topology

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The title

A robust approach to QUBO on the chimera topology

The plan

- Bit graph topologies (“chimera topology”)
- QUBO problem statement (“QUBO”)
- Some applications
- Obstacles for hardware
- Understanding the hardware
- Our reference solution (“robust approach”)
- Scaling up

Naming

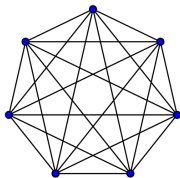
- V — set of $n := |V|$ variables (vertices, qubits).
- K_V — complete graph on V .
- $E \subseteq K_V$ — topology of edges (couplings).
- $S := \{0, 1\}^n$ — bit states.

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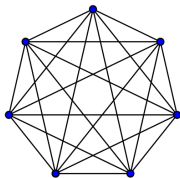
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Conventions

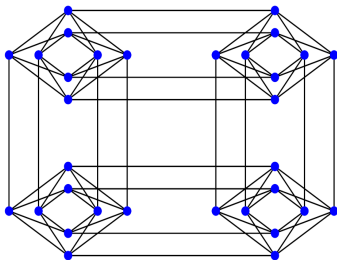
- $V = \{0, 1, \dots, n-1\}$.
- $E = K_n := K_V$ — complete graph.
- $E = C_k$ — Chimera graph.
- $E = C_{k,\ell}$ — Chimera fragment.
- $E = P_k$ — Pegasus graph.



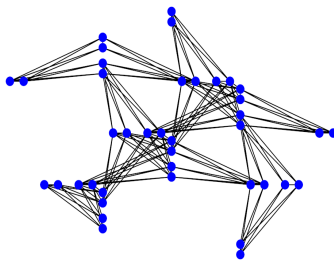
Complete graph K_7



Complete graph K_7



Chimera graph C_2



Pegasus graph P_2

Weights

- $b : V \rightarrow \mathbb{R}$ — variable biases.
- $c : E \rightarrow \mathbb{R}$ — coupling weights.
- $\beta \in \mathbb{R}_{>0}$ — Boltzmann distribution parameter.

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State functions

- $\mathcal{E} : S \rightarrow \mathbb{R}$ — state energy,

$$\mathcal{E}(x) := \sum_{\alpha \in V} b(\alpha)x_{\alpha} + \sum_{\{\alpha, \beta\} \in E} c(\{\alpha, \beta\})x_{\alpha}x_{\beta}.$$

- $W : S \ni x \mapsto e^{-\beta \mathcal{E}(x)} \in \mathbb{R}_{>0}$ — state weight.
- $Z := \sum_{x \in S} W(x)$ — partition function.
- $p : S \ni x \mapsto W(x)/Z \in (0, 1]$ — Boltzmann probability distribution.

Boltzmann distribution weights

- $W(x) := e^{-\beta \mathcal{E}(x)}$.
- Strongly prefers low energy states when sampling.
- May or may not find a global minimum.
- Used for Quantum Boltzmann Machines.

Some applications

Combinatorial problems

- Potentially lowest value of energy attained \iff Boolean formula satisfied.
- SAT.
- Factorization.

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Optimization problems

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Boltzmann machines

“Here are some cats. Show me another cat.”

Obstacles for hardware

Noisiness

- Low-order weights ignored in practice.
- No official data — needs experimental research.

Obstacles for hardware

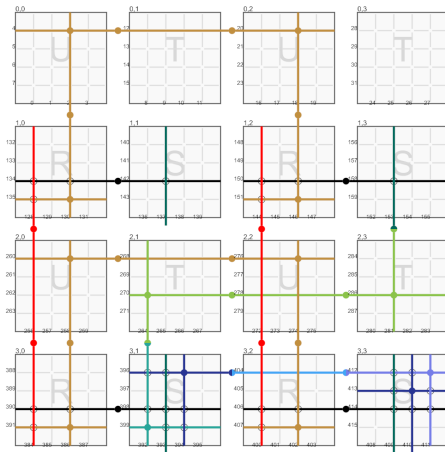
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Topology

- Most problems cannot be directly embedded in target topology.
- Bit chaining required.

Bit chaining



A $2\text{-bit} \times 2\text{-bit} \mapsto 4\text{-bit}$ factorization/multiplication circuit.
Embedded into C_4 — colors correspond to logical bits.

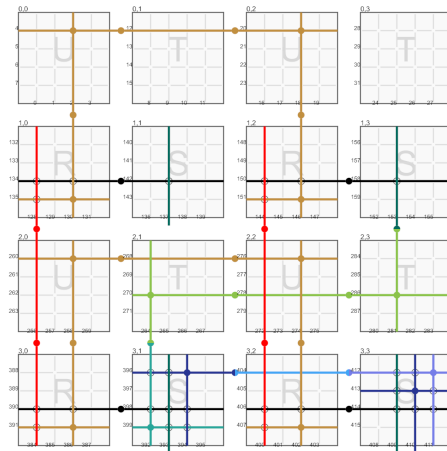
Problems with bit chaining

- Low-priority chaining \implies solutions cannot be mapped to logical bits.
- High-priority chaining \implies non-solutions (noisiness).
- Strongly limits the number of bits e.g. 2048-bit C_{16} can embed at most a 65-bit clique.
- Attempts at hardware-level chaining to counter these issues (Pegasus).

Questions

- How close to the ideal Boltzmann distribution are they?
- How close to the minimum energy they get?
- How noisy are they in general?

Let us prove that 3 is prime



A 2-bit \times 2-bit \mapsto 4-bit factorization/multiplication circuit.

Hardware sampling depends on more than energy

Factorize 3 by sampling the circuit 1000 times

Attempt	$1 \times 3 (\mathcal{E}_{\min})$	$3 \times 1 (\mathcal{E}_{\min})$	failed ($> \mathcal{E}_{\min}$)
1	444	14	542
2	387	61	552
3	307	78	615
4	406	31	563
5	282	96	622
6	240	64	696
7	278	52	670
8	159	117	724
9	190	174	636
10	439	49	512

Hardware sampling depends on more than energy

Use 10 rounds of random bit flips

Attempt	$1 \times 3 (\mathcal{E}_{\min})$	$3 \times 1 (\mathcal{E}_{\min})$	failed ($> \mathcal{E}_{\min}$)
1	189	65	746
2	193	67	740
3	230	90	680
4	160	91	749
5	225	54	721
6	232	111	657
7	280	60	660
8	162	76	762
9	166	90	744
10	204	96	700

Attaining minimum energy

Tailored example

- 2001 bits.
- Minimum theoretical energy -1000 .
- 10000 samples.
- Sampled energy range $[-690, -538]$.
- Random bit flips do not help, either.

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However

- Not really a noisiness issue, as the theoretical energy distribution is binomial.
- Problem trivially solved by hybrid approaches involving local search.

Boltzmann distribution

- $W : S \ni x \mapsto e^{-\beta\mathcal{E}(x)} \in \mathbb{R}_{>0}$ — state weight.
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Precision of $\beta\mathcal{E}$

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Experiment

- Set variable bias to 0.
- Sample variable 10000 times.
- Repeat $10\times$ for 2000 qubits, resulting in 20000 frequencies.

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Result

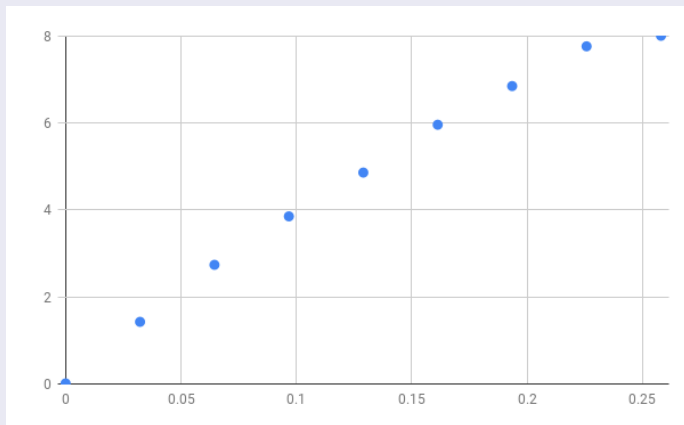
- Frequency ranging from 30% to 70%.
- Value of $\beta\mathcal{E} = 0$ interpreted as one between $[-0.8, 0.8]$.
- Resolution of $\beta\mathcal{E}$ seems to be even integers, but no finer.

Experiment

- Pick some bias values (b_1, \dots, b_m) spanning $[0, 1]$.
- Determine experimental probability p_j of getting value 1 by sampling multiple qubits with bias p_j , $j = 1, \dots, m$ (around half a million samples per j).
- Throw away outliers with probabilities too close to 0 or 1.
- As $p_j \simeq 1 - 1/(1 + e^{-\beta b_j})$, determine the experimental value y_j of βb_j by putting $y_j := -\ln(1/(1 - p_j) - 1)$.
- Plot y_j as a function of b_j and use linear regression to determine β .

Determining β

Result



Value of $y_j \simeq \beta b_j$ as a function of b_j . Experimentally, $\beta \simeq 32$.

Experiment

- Generate a QUBO instance on $C_{6,16}$ choosing biases and couplings uniformly among even numbers in $[-30, 30]$.
- Compute the minimum energy value and its probability using the exact reference solver.
- Sample 100 thousand points from the hardware implementation.

Experiment

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Result

- The exact minimum energy of -12000 has probability ~ 0.09 in the Boltzmann distribution, assuming the value of $\beta = 32$ applied to the normalized biases.
- Sampling 100 thousand points failed to produce the minimum energy value. The minimum energy attained was -11992 .

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Conclusions

Questions

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Answers

- Even in random cases, the probabilities seem to be off by many orders of magnitude.
- Sometimes the minimum is not found, even if the theoretical probability guarantees that.
- The noise of $\beta\mathcal{E}$ seems to be on the level of ± 0.8 , but rounding βE to even integers still produces results that we cannot explain.

Our QUBO exact reference solver

Methods

- Dynamic programming on bounded treewidth graphs.
- Cannot do full C_{16} .
- Uses some ideas by Alex Selby (minimum energy value).
- Computes much more using our new approach.

Our QUBO exact reference solver

Current state for $C_{k,\ell}$ ($k, \ell \in \mathbb{N}$, $k \leq \ell$)

Capability	CPU	GPU
Maximum k	7	8
Typical runtime	~ 1 minute	~ 10 seconds
Minimum energy value	$\mathcal{O}(\ell 2^{4k})$	
Minimum energy probability	—	$\mathcal{O}(\ell 2^{4k})$
Minimum energy state	$\mathcal{O}(\ell 2^{4k})$	$\mathcal{O}(\ell^2 2^{4k})$
Boltzmann sample	—	$\mathcal{O}(\ell^2 2^{4k})$
m Boltzmann samples	$\mathcal{O}(k\ell m + \ell^2 2^{4k})$	

Our QUBO exact reference solver

4x16 Chimera QUBO Solver - demo version

About us
Our Work
Our Code
QUBO Solver
Our Team
Career
Contact

```
512 112
0 0 0.3870967741935484
1 1 0.706174193548387
2 2 0.2550645112903225
3 3 0.4193548387096774
4 4 0.9354838709677419
5 5 0.06122510064518129
6 6 0.8954838709677419
7 7 0.3870967741935484
8 8 0.3225806451012903
9 9 0.006451012903225806
10 10 0.674193548387096
11 11 0.4483870967741895
12 12 0.5806451012903225
13 13 0.25806451012903225
14 14 0.96741935483871
15 15 0.2493258064518129
128 128 0.5161290322580645
129 129 0.16129032258064516
130 130 0.4819320967741935
```

Submit

Energy:

-10.129032135009765

Qubits:

{ 0:0, 1:1, 2:0, 3:0, 4:0, 5:0, 6:1, 7:0, 8:0, 9:0, 10:0, 11:0, 12:1, 13:1, 14:0, 15:0, 128:1, 129:1, 130:0, 131:0, 132:1, 133:0, 134:1, 135:1, 136:1, 137:1, 138:1, 139:1, 140:0, 141:1, 142:0, 143:1 }

USAGE

A QUBO problem description should be composed of an optional header:
(number_of_qubits) (number_of_lines)

Followed by a content with a list of node ids and theirs energy-values:
(node_id) (node_id) (value)

Example

```
512 8
0 0 -1
3 3 -2
4 4 1
7 7 2
0 4 -1
3 4 -0.5
0 7 0.5
3 7 1
```

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Online QUBO solver demo:
finds minimum energy value and state on $C_{4,16}$.

- Our exact solver does not scale that well.
- It can be used to validate hardware platforms or other heuristics.
- Biggest hope lies in hybrid local-search or MCMC solutions.

Thank you for your attention