Learning-based error mitigation for near-term quantum computers.

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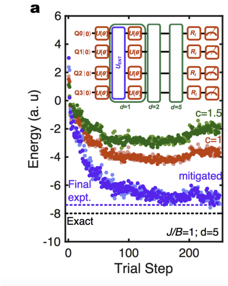
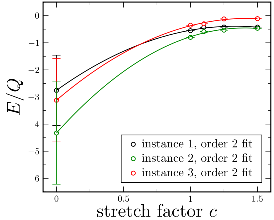
The outline

- Near-term quantum computing and error mitigation.
- Learning-based error mitigation.
- Enhancing power of error mitigation.
- Improving resource efficiency of error mitigation.

- Near-term quantum computers are seriously affected by noise.
- To obtain quantum advantage we need to mitigate effects of the noise.
- Error mitigation is challenging for large and deep quantum circuits necessary for quantum advantage.



Kandala et. al, Nature (2019)



Error mitigation with Clifford quantum-circuit data

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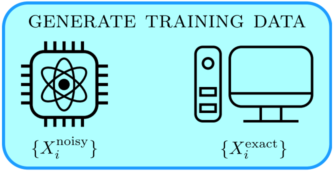
- X - an observable of interest, $|\psi\rangle$ - a quantum circuit of interest.
- $X_\psi^{\text{exact}} = \langle\psi|X|\psi\rangle$, X_ψ^{noisy} - the noisy expectation value.

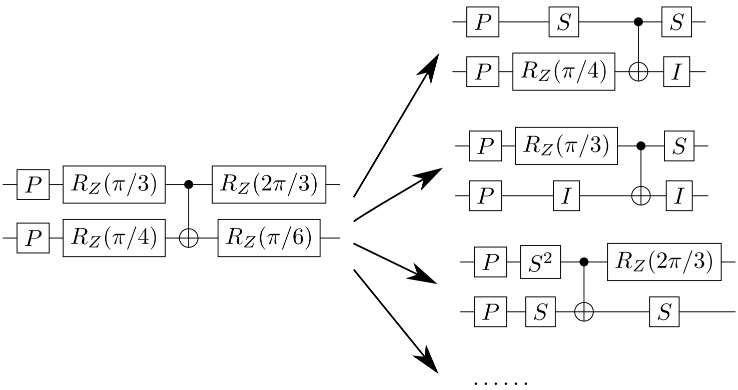
1. Choose near-Clifford classically simulable training circuits $\mathcal{S}_\psi = \{|\phi_i\rangle\}$.



Training circuits

- Near-Clifford circuits with up to $N = 50 - 80$ non-Clifford gates can be simulated classically.
- Replace non-Clifford gates by Clifford gates.
- An algorithm for IBM:

$$R_Z(\alpha) \to S^n, \quad S = R_Z(\pi/2), \quad n = 0, 1, 2, 3.$$



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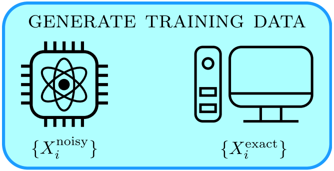
2. Construct a training set $\mathcal{T}_\psi = \{(X_{\phi_i}^{\text{noisy}}, X_{\phi_i}^{\text{exact}})\}$.



Clifford Data Regression (CDR)

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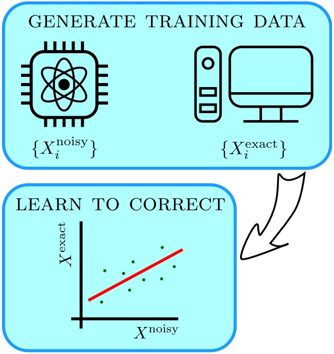
2. Construct a training set $\mathcal{T}_\psi = \{(X_{\phi_i}^{\text{noisy}}, X_{\phi_i}^{\text{exact}})\}$.
3. Learn an ansatz for X^{exact}:

$$X^{\text{exact}} = a_1 X^{\text{noisy}} + a_2,$$

$$\underset{a_1, a_2}{\operatorname{argmin}} \sum_{\phi_i \in \mathcal{T}_\psi} (X_{\phi_i}^{\text{exact}} - a_1 X_{\phi_i}^{\text{noisy}} - a_2)^2.$$

   - Corrects perfectly global depolarizing noise

$$\rho \Longrightarrow (1-p)\rho + p\mathbb{1}/(\dim\mathcal{H})$$



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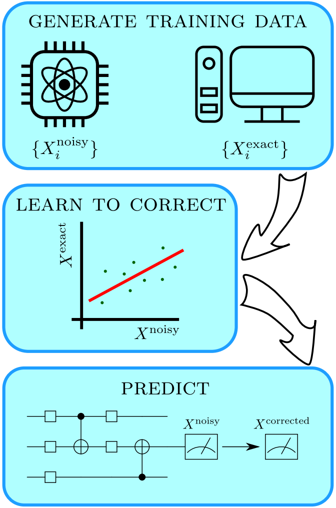
4. Use the ansatz to correct X_ψ^{noisy}.

$$X_\psi^{\text{exact}} = a_1 X_\psi^{\text{noisy}} + a_2.$$



QAOA for the quantum Ising model

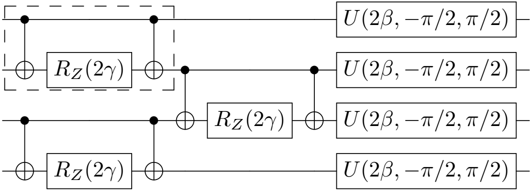
- The problem: Ground state simulation of a transverse-field 1D quantum Ising model ($g = 2$).

$$H = -g \sum_{j} X_j - \sum_{\langle j,j' \rangle} Z_j Z_{j'}$$

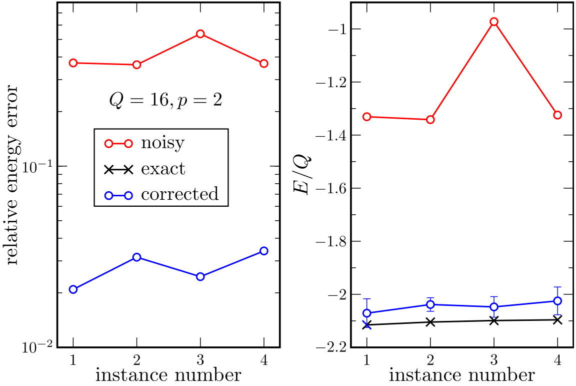
- The QAOA (Quantum Alternating Operator Ansatz) ansatz:

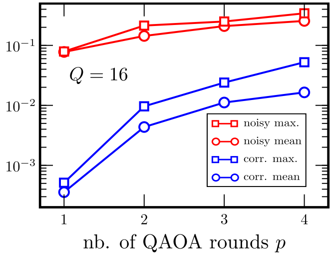
$$H = H_1 + H_2, \; H_2 = -g \sum_{j} X_j, \; H_1 = -\sum_{\langle j,j' \rangle} Z_j Z_{j'},$$

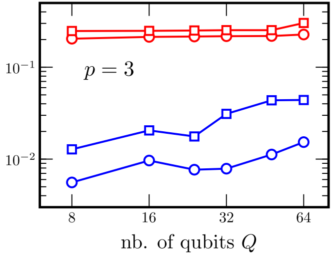
$$|\psi(\beta_1, \gamma_1, \ldots, \beta_p, \gamma_p)\rangle = \prod_{j=p,p-1\ldots,1} e^{i\beta_j H_2} e^{i\gamma_j H_1} (|+\rangle)^{\otimes Q}, \quad |+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle).$$

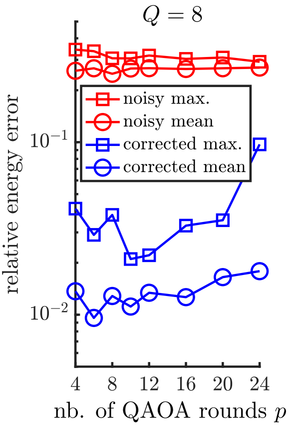


- The QAOA ansatz is optimized to find the ground state of the Quantum Ising model.
- Local minima of the optimization are simulated with IBM's Almaden quantum computer.
- A factor of 15 improvement obtained (60 CNOTs, 8 layers of CNOTs).









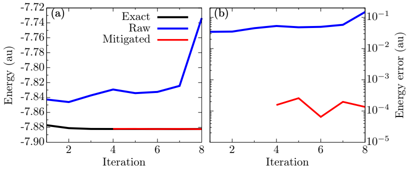
$$G^{\alpha} = \alpha G^{\text{noisy}} + (1-\alpha) G^{\text{exact}} \qquad \alpha = \frac{4}{p}$$

At least a factor of 10 improvement obtained.

Variational Quantum Eigensolver with Reduced Circuit Complexity

Yu Zhang,[1,*] Lukasz Cincio,[1] Christian F. A. Negre,[1] Piotr Czarnik,[1] Patrick Coles,[1]
Petr M. Anisimov,[2] Susan M. Mniszewski,[3] Sergei Tretiak,[1,4] and Pavel A. Dub[5]

- Variational quantum eigensolvers (VQE) search for ground states of quantum many-body systems minimizing energies of states encoded by parametrized quantum circuits.
- In collaboration with P. Dub (LANL) we applied CDR to benchmark potential of a novel VQE algorithm.
- We obtained orders of magnitude improvement of energies for a benchmark application of LiH ground state simulations.



CDR mitigation for VQE simulations of LiH molecule with IBM Bogota.

Unified approach to data-driven quantum error mitigation

Angus Lowe,[1,*] Max Hunter Gordon,[2,*] Piotr Czarnik,[3]
Andrew Arrasmith,[3] Patrick J. Coles,[3,4] and Lukasz Cincio[3,4]

- Zero Noise Extrapolation (Temme, Bravyi, Gambetta - 2017) increases the noise strength in a controlled manner to perform an extrapolation to the zero noise limit:

$$\lambda_j = c_j \lambda_0, \quad 1 = c_0 < c_1 < \cdots < c_n.$$

- Richardson extrapolation has an error $\mathcal{O}(\lambda_0^{n+1})$:

$$X^{\mathrm{Richardson}} = \sum_{j=0}^{n} X^{\mathrm{noisy},j} \gamma_j, \tag{1}$$

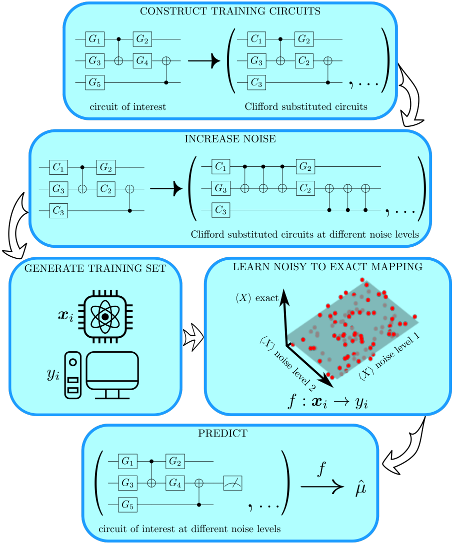
$$\sum_{j=0}^{n} \gamma_j = 1, \quad \sum_{j=0}^{n} \gamma_j c_j^k = 0 \ \text{ for } k = 1, \ldots, n.$$

- Find coefficients of the Richardson ansatz (1) from near-Clifford circuits similar to the circuit of interest.

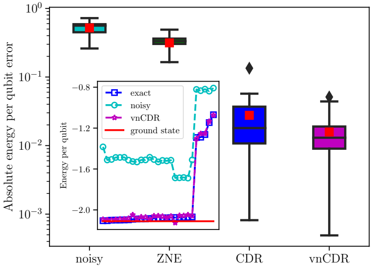
1. Choose training circuits as in the case of CDR.
2. Multiply noise level $j = 1, \ldots, n$ as in the case of Zero Noise Extrapolation (ZNE).
3. Construct a training set $\mathcal{T}_\psi = \{X^{\text{noisy},j}_{\phi_i}, X^{\text{exact}}_{\phi_i}\}$.
4. Learn a model for X^{exact}:

$$X^{\text{exact}} = \sum_j a_j X^{\text{noisy},j}.$$

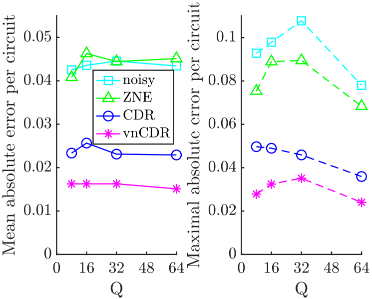
5. Use the model to correct $X^{\text{noisy},j}_{\psi}$.



vnCDR benchmarks



The ground state of a transverse Ising model, 8-qubit circuits, an IBM Ourense noise model.



Random quantum circuits, Q qubits, an IBM Ourense noise model.

- The new method (vnCDR) improves on performance of CDR and ZNE.

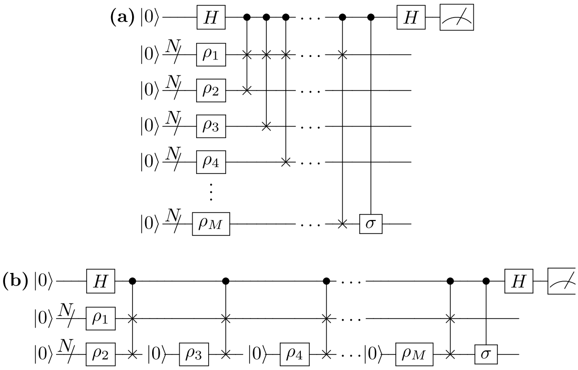
- Uses M copies of a noisy state ρ to "distill" a purified approximation of the exact one

$$X_M = \frac{\mathrm{Tr}[\rho^M X]}{\mathrm{Tr}[\rho^M]}.$$

- Exponentially suppresses incoherent errors

$$\rho = \sum_{i=0}^{2^Q-1} \lambda_i |\psi_i\rangle\langle\psi_i|,$$

$$X_M = \frac{\langle\psi_0|X|\psi_0\rangle}{1+\sum_{i=0}^{2^Q-1}(\lambda_i/\lambda_0)^M} + \frac{\sum_{i=1}^{2^Q-1}(\lambda_i/\lambda_0)^M\langle\psi_i|X|\psi_i\rangle}{1+\sum_{i=0}^{2^Q-1}(\lambda_i/\lambda_0)^M}.$$



Koczor (2020), Czarnik, Arrasmith, Cincio, Coles (2020)

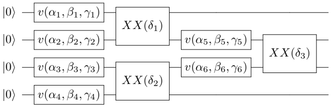
Unifying and benchmarking state-of-the-art quantum error mitigation techniques

Daniel Bultrini,[1,2,*] Max Hunter Gordon,[3,*] Piotr Czarnik,[1]
Andrew Arrasmith,[1,4] Patrick J. Coles,[1,4] and Lukasz Cincio[1,4]

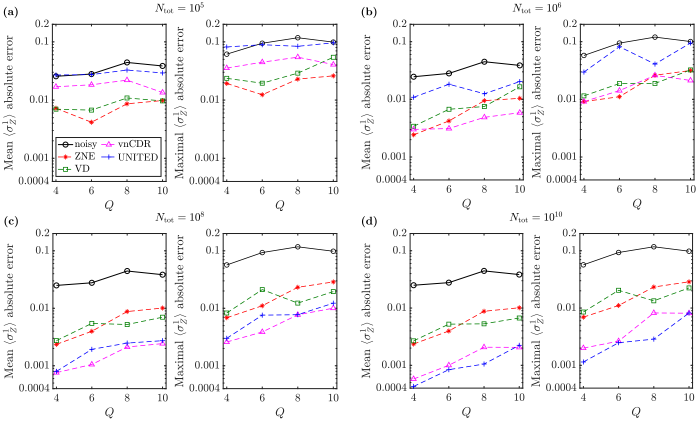
1. Choose near-Clifford training circuits as for CDR.
2. Boost state preparation noise as for ZNE and vnCDR.
3. Perform VD on the training circuits.
4. Construct a training set $\mathcal{T}_\psi = \{X_{\phi_i}^{jM}, X_{\phi_i}^{\text{exact}}\}$.
5. Learn a model for X^{exact}:

$$X^{\text{exact}} \approx \sum_{j=0}^{n} \sum_{M=1}^{M_{\max}} a_{jM} X_{jM}.$$

6. Use the model to correct X_ψ^{jM}.

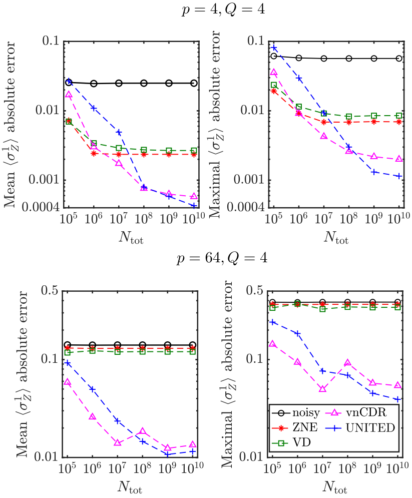


$$v(\alpha,\beta,\gamma) = R_Z(\alpha)R_Y(\beta)R_Z(\gamma)$$



More general ansaetze result in better "performance ceilings".

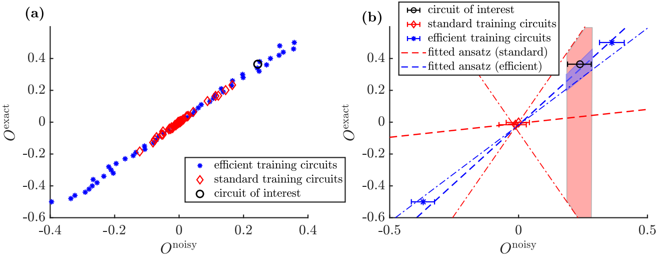
Benchmarking advanced approaches



With better "performance ceilings" more shots are required to reach it.

Trapped-ion noise model: Colin J Trout et al 2018 New J. Phys. 20 043038

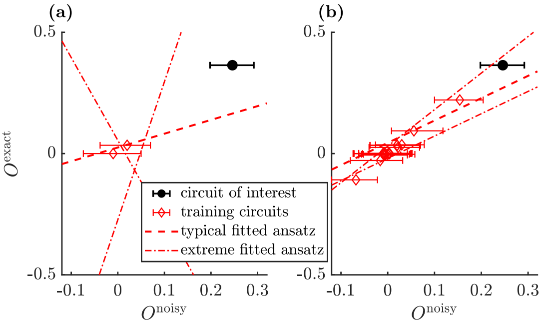
Improving CDR shot-efficiency



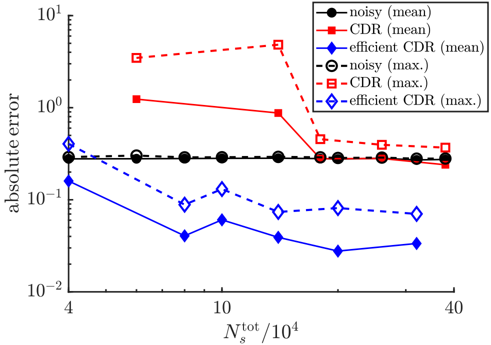
The ground state of 8-qubit XY model - clustering of the training circuits and its effects on the quality of CDR error mitigation.

$$H = \sum_{i=1}^{Q-1} (X_i X_{i+1} + Y_i Y_{i+1}) + X_Q X_1 + Y_Q Y_1. \quad (2)$$

$$O_1 = X_1 X_{Q/2+1}, O_2 = X_2 X_{Q/2+2}, \ldots, O_{Q/2} = X_{Q/2+1} X_Q,$$
$$O_{Q/2+1} = Y_1 Y_{Q/2+1}, \ldots, O_N = Y_{Q/2+1} Y_Q,$$
$$\langle O_1 \rangle = \langle O_2 \rangle = \ldots \langle O_Q \rangle.$$



The ground state of 8-qubit XY model (2). Translational symmetry and the Hamming weight preservation (U(1) symmetry) can be utilized to improve efficiency.



Error for the half-chain correlators of the ground state of 6-qubit XY model (60 CNOTs, 20 layers of CNOTs) and IBM Toronto plotted versus total error mitigation shot-cost.

Conclusions

- Effects of the noise on the expectation values can be learned from training circircuits similar to the circuit of interest.
- Including additional information about noise effects enables better quality of the learning.
- Preventing clustering of the training data is crucial for learning with limited shot resources.
- Up to orders of magnitude improvement of results quality demonstrated with real-world devices.