Quantum Switch as Quantum Circuit – Theory and Practise

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Outline

(I) Introduction

1. the role of switch,
2. the basic definition,
3. more theoretical approach to swapping information.

(II) Quantum Switch

1. the basic circuits,
2. a decomposition into basic set of gates.

(III) The entanglement level

1. a behaviour of the level of entanglement as invariant,
2. the level of entanglement without and with noise presence.

(IV) Summary

1. further works,
2. remarks.

Introduction

(switch, definition, and some theory)

The role of quantum switch

In points, the role, possible application or usefulness of quantum switch (QS) can summarised as:

- information switching and swapping seem to be fundamental elements of quantum communication protocols,
- another crucial issue is a presence of entanglement and its level in inspected quantum systems, especially in QS,
- the QS is not complicated, therefore can be used to show how the noise influence on its work,
- the QS is a quite basic block then can be used as benchmark for quality of present quantum technology,
- the QS can be used as an example of device which we want to debug with the use of other quantum circuit.

The basic definition of Quantum Switch – 1/2

The idea of a quantum switch can shown with three-qubit controlled swap gate. Initial state quantum system is as:

$$|\Psi_{qs}\rangle = |A\rangle|B\rangle|C\rangle, \tag{1}$$

where the first qubit $|A\rangle$ and the second one $|B\rangle$ are unknown quantum qubit states:

$$|A\rangle = \alpha_0|0\rangle + \beta_0|1\rangle, \quad |B\rangle = \alpha_1|0\rangle + \beta_1|1\rangle, \tag{2}$$

where $\alpha_0, \beta_0, \alpha_1, \beta_1 \in \mathcal{C}$ and $|\alpha_0|^2 + |\beta_0|^2 = 1$, $|\alpha_1|^2 + |\beta_1|^2 = 1$.
The third qubit $|C\rangle$ is a controlling qubit and it accepts one of two states $|0\rangle$ or $|1\rangle$. In general, QS can be regarded as a controlled SWAP gate. The mentioned gate swaps the states $|A\rangle$ and $|B\rangle$ according to a state of the qubit $|C\rangle$.

The basic definition of Quantum Switch – 2/2

The way of operating for the quantum switch may be described by two cases. The first case takes place when the state of the qubit $|C\rangle$ is $|0\rangle$:

$$|A\rangle|B\rangle|0\rangle \Rightarrow |A\rangle|B\rangle|0\rangle. \tag{3}$$

The quantum switch does not swap the states of input qubits. The operation of swapping is connected with the second case when the state of the qubit $|C\rangle$ is $|1\rangle$:

$$|A\rangle|B\rangle|1\rangle \Rightarrow |B\rangle|A\rangle|1\rangle, \tag{4}$$

as it can be seen in Eq. (4), the states of qubits $|A\rangle$ and $|B\rangle$ were swapped.

Swapping information

We consider two-partite system A and B consisting of a d-dimensional qudits. The corresponding space of states of the system is denoted as

$$E(\mathcal{H}^A \otimes \mathcal{H}^B) = \{Q : Q \geqslant 0 \text{ and } \mathrm{Tr}\,(Q) = 1\},\ \mathcal{H}^A \approx \mathcal{H}^B \simeq \mathbb{C}^d. \tag{5}$$

Next, With the qudits A and B, we associate the corresponding observers which for a given global quantum state Q have to their disposal only information contained on the corresponding reduced density matrices (termed RDMs) defined usually as

$$Q^A = \mathrm{Tr}_{\mathrm{B}}\,(Q)\,, \tag{6}$$

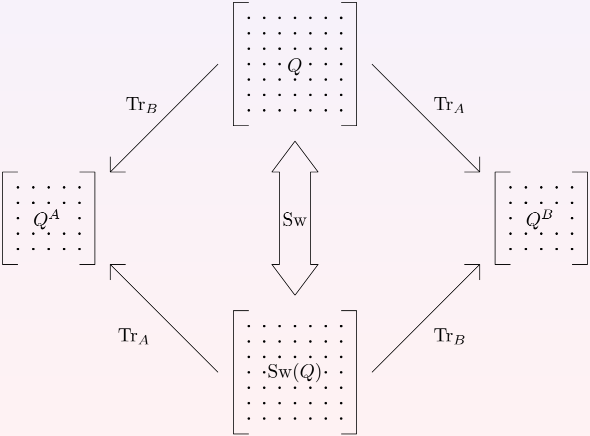
for observer O^A and

$$Q^B = \mathrm{Tr}_{\mathrm{A}}\,(Q)\,, \tag{7}$$

for observer O^B.

SWAP operation – 1/2

A graphical representation of the SWAP operation. Local information exchange is performed on the initial state Q (the final state, after the operation, is $\mathrm{Sw}(Q)$). One can observe the swapping of subsystems A and B what may be calculated by the operation of partial trace.



SWAP operation – 2/2

Definition

An operation Sw:

$$\mathrm{Sw} : E(\mathcal{H}^A \otimes \mathcal{H}^B) \to E(\mathcal{H}^B \otimes \mathcal{H}^A), \tag{8}$$

will be called a swapping local information operation for a given $Q \in E(\mathcal{H}^A \otimes \mathcal{H}^B)$ if the following equalities are true:

$$\mathrm{Tr}_\mathrm{B}\left(\mathrm{Sw}(Q)\right) = Q^B \text{ and } \mathrm{Tr}_\mathrm{A}\left(\mathrm{Sw}(Q)\right) = Q^A. \tag{9}$$

Remark

Let us emphasise once more that swapping operations play a very important role in the constructions of the so-called quantum repeaters which seems to be one of the important ingredients of long-distance quantum networks for quantum key distributions protocols realisation.

The following proposition can be used to calcuate the level of entanglement during the information switching.

Proposition – Easy Separabiliy Criterion (ESC)

Let $Q \in E(\mathbb{C}^d \otimes \mathbb{C}^d)$ be a separable state. Then

$$-\sum_{j=1}^{d^2} \lambda_j \left(\sum_{\alpha=1}^{d} (\tau_j^\alpha)^2 \log(\tau_j^\alpha)^2) \right) \leqslant -\sum_{j=1}^{d^2} \lambda_j \log \lambda_j. \qquad (10)$$

Remark

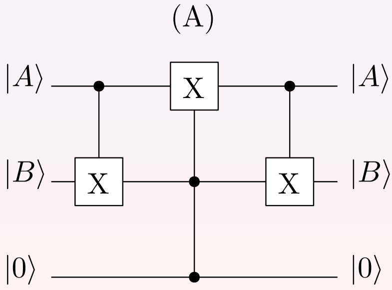
The presented separability criterion in the case of pure states is exact. However, in general case of mixed states its domain of effective action is rather week comparing with many other separability versus non-separability criteria known in the literature. The advantage of the inequality Eq. 10 is that it refers only to the basic, spectral data of a state under consideration.

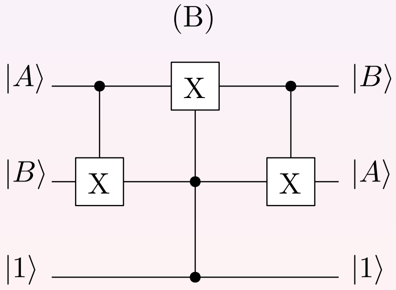
Quantum Switch

(switch as circuit, decomposition)

Quantum switch basic circuit – 1/2

The circuits illustrating the operation of the quantum switch for qubits. If the state of the controlling qubit is $|0\rangle$ (case (A)) the switch does not change the order of first two input states. When the state of the third qubit is expressed as $|1\rangle$ (case (B)), the quantum switch swaps the input states $|A\rangle$ and $|B\rangle$. The matrix (C) represents the unitary operator of the switch operation.





Quantum switch basic circuit – 2/2

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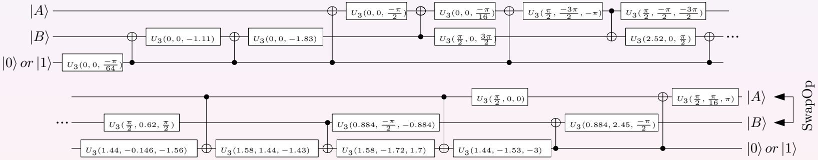
(C)

$$U_{qs} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

It should be underlined that the operation U_{qs} captures the complete working of the switch.

Decomposition of $U_{qs}(t)$ operation for $t = 1/4$. The set of used gates enables the circuit's implementation in qiskit package and quantum machine IBM Q. Decompositions may be realised for arbitrary t what requires the changes in values of rotating gates U3 parameters.



Now, we define a simplified Hamiltonian's form, because we can directly take advantage of the fact that the switch realises swap operation only if the third qubit is in the state $|1\rangle$. That leads to the basic direct form of the Hamiltonian, describing the dynamics of the operation performed by the switch, where we reuse the Pauli X and Z operators applied in a subspace of the first and the second qubit with two additional couplings equal $\frac{1}{2}$.

$$H_{qs} = \frac{1}{2}\left(|011\rangle\langle 101| + |101\rangle\langle 011|\right) - \frac{1}{2}\left(|011\rangle\langle 011| + |101\rangle\langle 101|\right) = \left(\begin{array}{cccccccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right). \tag{11}$$

Such approach can be used to the evaluation of entanglement level during the switch work.

The Hamiltonian H_{qs} together with a time variable t allows us to express the dynamics of the switch as a unitary time evolution operator:

$$U_{qs}(t) = e^{-\mathbb{i} t H_{qs}}, \tag{12}$$

and $\mathbb{i}$ represents the imaginary unit value.
A matrix form of the operator, for real values of t variable, is:

$$U_{qs}(t) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1+e^{(\mathbb{i}\pi t)}}{2} & 0 & \frac{1-e^{(\mathbb{i}\pi t)}}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1-e^{(\mathbb{i}\pi t)}}{2} & 0 & \frac{1+e^{(\mathbb{i}\pi t)}}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \tag{13}$$

where $t \in \langle 0, 1 \rangle$, and for $t = 1$ the switch correctly realises the swap operation for input states.

If the unitary operation Eq. (13) is used then the system's state (with the control qubit in the state $|0\rangle$) may be expressed as:

$$U_{qs}(t)|\Psi_{qs0}^{U_{qs}(t)}\rangle = \begin{pmatrix} \alpha_0\alpha_1 \\ 0 \\ \alpha_0\beta_1 \\ 0 \\ \alpha_1\beta_0 \\ 0 \\ \beta_0\beta_1 \\ 0 \end{pmatrix}. \qquad (14)$$

In this case there is no swapping of input states.

The action of swap operation is performed when the state of control qubit is $|1\rangle$:

$$U_{qs}(t)|\Psi_{qs1}\rangle = |\Psi_{qs1}^{U_{qs}(t)}\rangle = \begin{pmatrix} 0 \\ \alpha_0\alpha_1 \\ 0 \\ \frac{1}{2}\Lambda^-(t)\alpha_1\beta_0 + \frac{1}{2}\Lambda^+(t)\alpha_0\beta_1 \\ 0 \\ \frac{1}{2}\Lambda^+(t)\alpha_1\beta_0 + \frac{1}{2}\Lambda^-(t)\alpha_0\beta_1 \\ 0 \\ \beta_0\beta_1 \end{pmatrix}, \tag{15}$$

where $t \in \langle 0, 1\rangle$ and $\Lambda^{\pm}(t) = \left(1 \pm e^{\mathbb{i}\pi t}\right)$.

In this case we perform the swap operation of input states.

A density matrix, for the above pure state, is defined as:

$$\rho_{qs1}(t) = |\Psi_{qs1}^{U_{qs}(t)}\rangle\langle\Psi_{qs1}^{U_{qs}(t)}|. \tag{16}$$

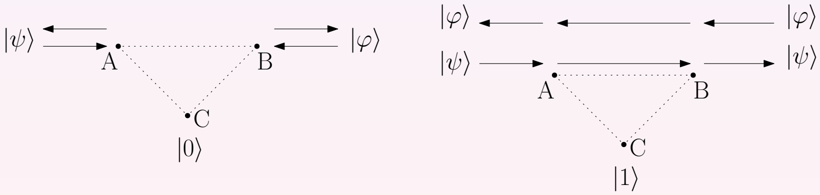
The entanglement level

The entanglement level

(the entanglement in switch circuit, behaviour, max value, compact formula)

The entanglement level

The examined entanglement level concerns mainly qubits A and B which exchange their states (expressed as $|\psi\rangle$ and $|\varphi\rangle$) if controlling qubit $C = |1\rangle$. The entanglement level may be also analysed between pairs: $A \ldots C$ and $B \ldots C$, what is marked above with the dotted line.



The entanglement level

We examine the entanglement between qubits A and B when C is in state $|1\rangle$. To do this, we utilise the Negativity criterion for two-qubit ρ state:

$$\mathcal{N}(\rho) = \frac{||\rho^{T_A}||_1 - 1}{2}, \tag{17}$$

where ρ^{T_A} represents a state after the partial transposition with respect to the first subsystem. The trace norm $||X||_1$:

$$||X||_1 = \mathrm{Tr}\left(|X|\right) = \mathrm{Tr}\left(\sqrt{X^\dagger X}\right). \tag{18}$$

Remark

The Negativity measure value can be calculated the absolute value of the sum of all negative eigenvalues λ_i of the operator ρ^{T_A}:

$$\mathcal{N}(\rho) = \sum_{\lambda_i<0} |\lambda_i| = \sum_i \frac{|\lambda_i| - \lambda_i}{2}. \tag{19}$$

The Negativity measure is used for a system of two qubits, so there are only four eigenvalues.

The Negativity criterion allows us to formulate the following theorem:

Theorem

Let t be a real number and $t \in \langle 0, 1 \rangle$ (closed interval). The quantum switch, expressed as the unitary operation (12) for the input state $|AB1\rangle$, introduces entanglement between qubits $|A\rangle$ and $|B\rangle$ for $t \in (0, 1)$ and there is no entanglement in moments $t = 0$ and $t = 1$.

Main elements of proof:

- the use of entanglement measure e.g. negativity,
- technical issues of algebraic transformation (but basic substitutions are enough),
- compact form of negativity for switch for two unknown inputs.

After using the Eq. 17, we calculate the following state:

$$\mathcal{N}(\rho_{qs}(t)) = \mathcal{N}\left(\begin{array}{cccc} a(a)^{\star} & a\,(G)^{\star} & a\,(H)^{\star} & a(b)^{\star} \\ (a)^{\star}\,(G) & (G)\,(G)^{\star} & (G)\,(H)^{\star} & (b)^{\star}\,(G) \\ (a)^{\star}\,(K) & (K)\,(G)^{\star} & (K)\,(H)^{\star} & (b)^{\star}\,(K) \\ b(a)^{\star} & b\,(G)^{\star} & b\,(H)^{\star} & b(b)^{\star} \end{array}\right), \qquad (20)$$

where $\rho_{qs}(t)$ symbolises a density matrix calculated as a partial trace operation which erases $|C\rangle = |1\rangle$ from the system (the density matrix describes only qubit states A and B).

We use the substitutions:

$$\begin{aligned} &a = \alpha_0\alpha_1, b = \beta_0\beta_1, c = \alpha_0\beta_1, d = \alpha_1\beta_0, \\ &G = c\left(\tfrac{1}{2} + \tfrac{1}{2}e^{i\pi t}\right) + d\left(\tfrac{1}{2} - \tfrac{1}{2}e^{i\pi t}\right), \\ &H = d\left(\tfrac{1}{2} + \tfrac{1}{2}e^{i\pi t}\right) + c\left(\tfrac{1}{2} - \tfrac{1}{2}e^{i\pi t}\right), \\ &K = c\left(\tfrac{1}{2} - \tfrac{1}{2}e^{i\pi t}\right) + d\left(\tfrac{1}{2} + \tfrac{1}{2}e^{i\pi t}\right). \end{aligned} \tag{21}$$

The partial transposition according to the qubit $|A\rangle$ must be done, and the density matrix takes form:

$$\rho_{U_{sq}}^{T_A B} = \begin{pmatrix} a(a)^\star & a\,(G)^\star & (a)^\star\,(K) & (K)\,(G)^\star \\ (a)^\star\,(G) & (G)\,(G)^\star & b(a)^\star & b\,(G)^\star \\ a\,(H)^\star & a(b)^\star & (K)\,(H)^\star & (b)^\star\,(K) \\ (G)\,(H)^\star & (b)^\star\,(G) & b\,(H)^\star & b(b)^\star \end{pmatrix}, \qquad (22)$$

the marking $\rho^{T_A B}$ tells that the system A is partially transposed, and the system B remains unchanged.

Naturally, such matrix has four eigenvalues, but only two of them (labelled as n_1 and n_2) may be negative numbers:

$$\begin{aligned}\lambda^{n_1} &= \tfrac{1}{4}\sqrt{-1+e^{2\mathbb{i}\pi t}}(\alpha_0\beta_1-\alpha_1\beta_0)\sqrt{\left(-1+e^{-2\mathbb{i}\pi t}\right)\left(\alpha_1^{\star}\beta_0^{\star}-\alpha_0^{\star}\beta_1^{\star}\right)^2},\\ \lambda^{n_2} &= \tfrac{1}{4}\sqrt{-1+e^{2\mathbb{i}\pi t}}(\alpha_1\beta_0-\alpha_0\beta_1)\sqrt{\left(-1+e^{-2\mathbb{i}\pi t}\right)\left(\alpha_1^{\star}\beta_0^{\star}-\alpha_0^{\star}\beta_1^{\star}\right)^2}.\end{aligned}$$

After further algebraic transformations, the value of the Negativity measure may be expressed as:

$$\mathcal{N}(\rho_{U_{qs}}(t)) = \sqrt{\frac{(\sin(t\pi))^2\,|\alpha_1\beta_0-\alpha_0\beta_1|^4}{4}}. \tag{23}$$

The above equation shows that the value of the Negativity measure is time-dependent.

The Eq. 23 clearly shows that the values of amplitudes are constant. It is also easy to point out in which moments the entanglement vanishes, because of the basic properties of the sin function, for $t = 0$ and $t = 1$ the value of the Negativity measure equals zero.

Corollary

For the initial state $|AA1\rangle$, i.e. states of the first two qubits are the same, Eq. 23 illustrates that there is no entanglement in the system, because expression $a_1 b_0 - a_0 b_1$ takes the form $a_0 b_0 - a_0 b_0$, and equals zero.

Remark

It should be emphasised that the level of entanglement is calculated for the system without distortions. The highest entanglement level, according to time t, is:

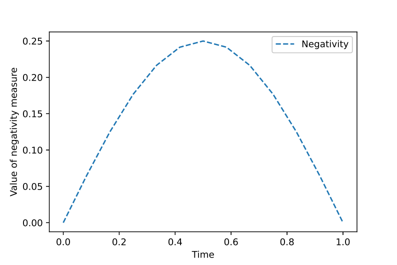
$$M_{Ent}(t) = \max_{t \in \langle 0,1 \rangle} \mathcal{N}(\rho_{U_{qs}}(t)) = \frac{1}{2}. \tag{24}$$

After the analysis of algebraic form of the Negativity value, we obtain that its maximum value appears in the moment $t = \frac{1}{2}$, what results from the basic properties of the *sin* function.

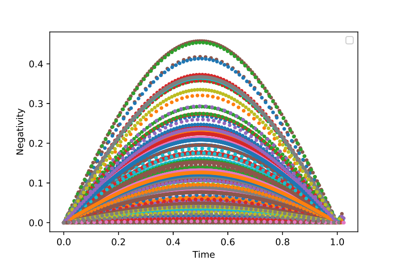
The numerical example – 1/4

The chart (A) depicts the changes of the entanglement level between qubits A and B during the switch operating in time $t \in \langle 0, 1 \rangle$. States $|A\rangle$, $|B\rangle$ are described respectively as: $|A\rangle = |+\rangle$, $|B\rangle = |0\rangle$. Whereas, the chart (B) shows the values of the Negativity measure for the transfer of arbitrary selected states (128 states were used to create the chart (B)).

(A)



(B)



In this case noise is not present.

The numerical example – 2/4

In the next numerical example an additional noise as the maximally mixed state is added to examined state:

$$\rho(t)_{qsmix} = (p \cdot \rho_{qs1}(t)) + (1-p)\frac{\mathbb{I}}{8}, \tag{25}$$

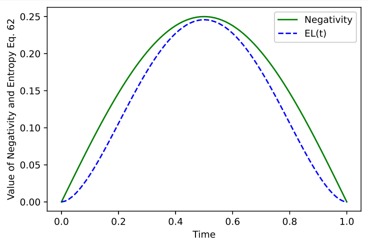
where $p \in \langle 0, 1 \rangle$ and $\mathbb{I}$ is the identity matrix sized 8×8. Unfortunately, when the extra noise is present, the criterion described in Proposition on slide 10 does not detect the entanglement properly. However, the left-hand side of the equation still indicates the changes in Entropy values because the switch is working (naturally, the value of Negativity measure points out the lack of entanglement for $p \leqslant \frac{1}{2}$).
The values for $EL(t)$ (based on Prop. ESC on slide 10) were calculated for $Q = \rho_{qs1}(t)$:

$$EL(t) = \left(-\sum_{j=1}^{d^2} \lambda_j \left(\sum_{\alpha=1}^{d} (\tau_j^{\alpha})^2 \log((\tau_j^{\alpha})^2) \right) \right) - \left(-\sum_{j=1}^{d^2} \lambda_j \log \lambda_j \right). \tag{26}$$

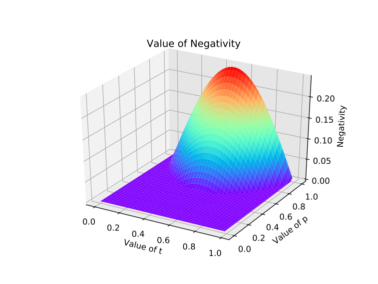
The numerical example – 3/4

Values of the Negativity measure (A) and values calculated by the criterion Proposition ESC on slide 10, Eq. 10 for the switch during its operating on states $|A\rangle = |+\rangle$, $|B\rangle = |0\rangle$. It is clear that both criteria evaluate the entanglement levels for pure states. The values of $EL(t)$ are obtained as the differences between both sides of inequality Prop. 10. It means that only for $t = 0$ and $t = 1$ both sides of the inequality are the same. In case (B), after the noise introduction (maximally mixed state), the Negativity measure still detects entanglement properly, but its level is decreased by the p value. Case (C) shows that $EL(t)$ can be also used to indicate the presence of distortions by generating the negative values

(A)



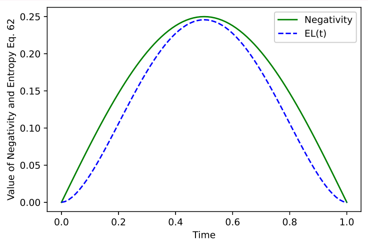
(B)



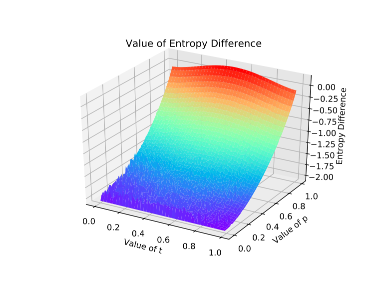
The numerical example – 4/4

Values of the Negativity measure (A) and values calculated by the criterion Proposition ESC on slide 10, Eq. 10 for the switch during its operating on states $|A\rangle = |+\rangle$, $|B\rangle = |0\rangle$. It is clear that both criteria evaluate the entanglement levels for pure states. The values of $EL(t)$ are obtained as the differences between both sides of inequality Prop. 10. It means that only for $t = 0$ and $t = 1$ both sides of the inequality are the same. In case (B), after the noise introduction (maximally mixed state), the Negativity measure still detects entanglement properly, but its level is decreased by the p value. Case (C) shows that $EL(t)$ can be also used to indicate the presence of distortions by generating the negative values

(A)



(C)



A more realistic noise – 1/2

We use the Dzyaloshinskii–Moriya interaction (DMI) [Dzyaloshinskii, 1958], [Moriya, 1960]. The applied interaction is described as:

$$H_{DM} = (\sigma_X^{(i)} \sigma_X^{(i+1)} - \sigma_Y^{(i)} \sigma_Y^{(i+1)}). \tag{27}$$

The marking $\sigma_X^{(i)}$ tells us that the qubits, indexed as i and $i+1$, may be affected by one of the Pauli operators: X or Y. We introduce an additional real-valued parameter $D_s \in \langle 0, 1 \rangle$ which describes the strength of the interaction. The mentioned parameter may be utilised directly:

$$H_{DM}(D_s) = D_s \cdot \left(\sigma_X^{(i)} \sigma_X^{(i+1)} - \sigma_Y^{(i)} \sigma_Y^{(i+1)} \right). \tag{28}$$

A more realistic noise – 2/2

To examine the DMI influence on the switch, we need a new Hamiltonian H_{TOT} which represents dynamics of these two joined systems:

$$H_{\mathrm{TOT}} = H_{qs} + D_s \cdot H_{\mathrm{DM}}, \tag{29}$$

where t stands for the time and D_s for the DMI strength.
Thus, we can construct a unitary operator U_{qs}^{DM} which is equivalent of H_{TOT}:

$$U_{qs}^{\mathrm{DMI}}(t, D_s) = e^{-\mathbb{i}(t \cdot H_{qs} + D_s \cdot H_{\mathrm{DM}})}. \tag{30}$$

It should be stressed that for $D_s = 0$, we obtain the operator describing only the switch's operating. The time variable t accepts values from the interval $\langle 0, 1 \rangle$.

However, the influence of DMI is modelled by the following relation, which describes the intrinsic decoherence effect, where the state in a moment t is given by the Milburn equation [Milburn, 1991]:

$$\rho(t)_{DMI} = \sum_{m,n} \exp\left(-\frac{\gamma t \pi}{2}(\mu_m - \mu_n)^2 - \mathbb{i}(\mu_m - \mu_n)t\pi\right) \cdot \langle \Xi_m | \rho(0) | \Xi_n \rangle | \Xi_m \rangle \langle \Xi_n |, \tag{31}$$

where μ_m, μ_n stand for eigenvalues and Ξ_m, Ξ_n for eigenvectors of H_{TOT} Hamiltonian. The symbol γ refers to the intrinsic decoherence rate. Eigenvalues of the Hamiltonian H_{TOT} take the form:

$$\begin{aligned} &\mu_0 = -1, \mu_1 = 0, \mu_2 = 0, \mu_3 = 0, \mu_4 = -2 \cdot D_s, \\ &\mu_5 = -2 \cdot D_s, \mu_6 = 2 \cdot D_s, \mu_7 = 2 \cdot D_s, \end{aligned} \tag{32}$$

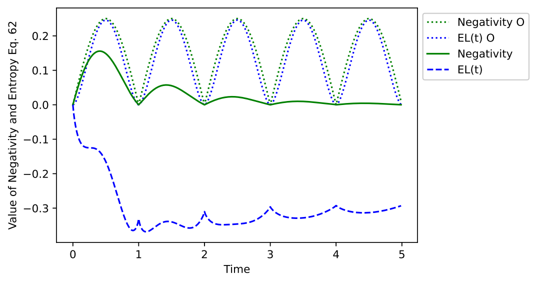
and its eigenvectors are:

$$\begin{aligned} &\Xi_0 = \tfrac{1}{\sqrt{2}}(-|3\rangle + |5\rangle), && \Xi_1 = \tfrac{1}{\sqrt{2}}(|3\rangle + |5\rangle), && \Xi_2 = |4\rangle, \\ &\Xi_3 = |2\rangle, && \Xi_4 = \tfrac{1}{\sqrt{2}}(-|1\rangle + |7\rangle), && \Xi_5 = \tfrac{1}{\sqrt{2}}(-|0\rangle + |6\rangle), \\ &\Xi_6 = \tfrac{1}{\sqrt{2}}(|1\rangle + |7\rangle), && \Xi_7 = \tfrac{1}{\sqrt{2}}(|0\rangle + |6\rangle). \end{aligned} \tag{33}$$

Switch & Noise – 1/2

Plot (A) shows values of Negativity and EL(t) for a pure state which dynamics is described by the operator U_{qs}^{DMI} without DMI distortions ($D_s = 0$). Plot (B) presents entanglement levels when DMI is present and $D_s = 0.25$ (again, the operator U_{qs}^{DMI} was used). On both graphs, the lines with the additional symbol O (Negativity O and EL(t) O) refer to cases without the intrinsic decoherence. Other lines depict the switch behaviour with the intrinsic decoherence for $\gamma = 0.5$.

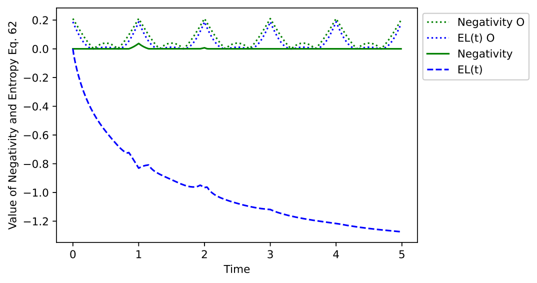
(A)



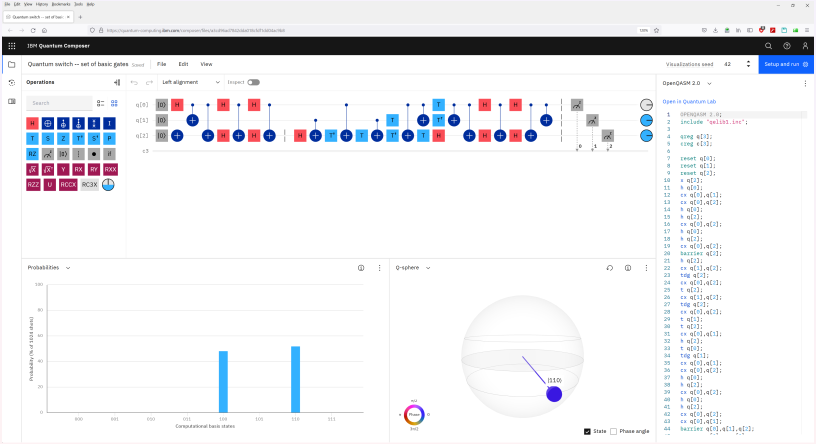
Switch & Noise – 2/2

Plot (A) shows values of Negativity and EL(t) for a pure state which dynamics is described by the operator U_{qs}^{DMI} without DMI distortions ($D_s = 0$). Plot (B) presents entanglement levels when DMI is present and $D_s = 0.25$ (again, the operator U_{qs}^{DMI} was used). On both graphs, the lines with the additional symbol O (Negativity O and EL(t) O) refer to cases without the intrinsic decoherence. Other lines depict the switch behaviour with the intrinsic decoherence for $\gamma = 0.5$.

(B)

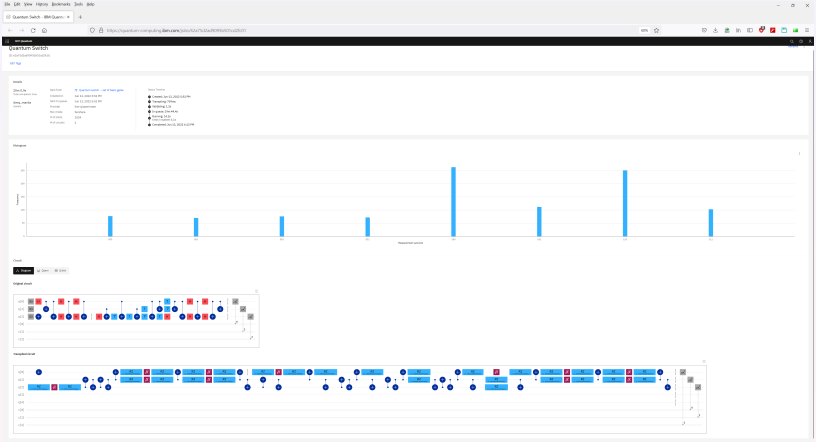


IBM Q – preparations



IBM Q – simulation

IBM Q – Manila


File Edit View History Bookmarks Tools Help
Quantum Switch - IBM Quantum
Quantum Switch
Details
Histogram
Circuit
Original circuit
Transpiled circuit

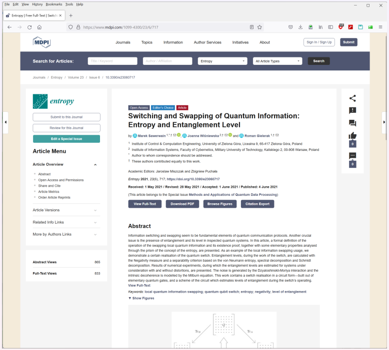
Summary

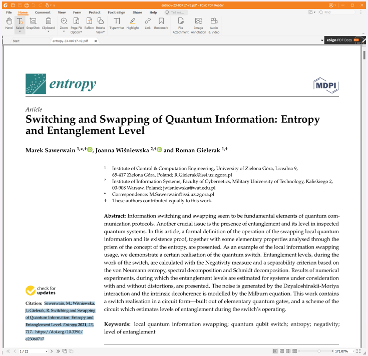
Conclusions and some tasks for further work:

- more experiments at IBM Q, tuning quality for switch circuit,
- experiments in IBM Metal API, such approach should allow us to achieve better accuracy,
- switch circuit is good candidate for benchmark tool of quality for current quantum technology,
- switch for multi-qubits states, switch for qudits,
- other noise models can be checked.

Thank you for your attention!

Presentation is based on our paper:





- Sawerwain, M.; Wiśniewska, J.; Gielerak, R. Switching and Swapping of Quantum Information: Entropy and Entanglement Level. Entropy 2021, 23, 717. `https://doi.org/10.3390/e23060717`