Fast spin initialization of an electron trapped in a gated nanowire quantum dot

Stanisław Bednarek
Michał Górski
Jarosław Pawłowski
Grzegorz Skowron

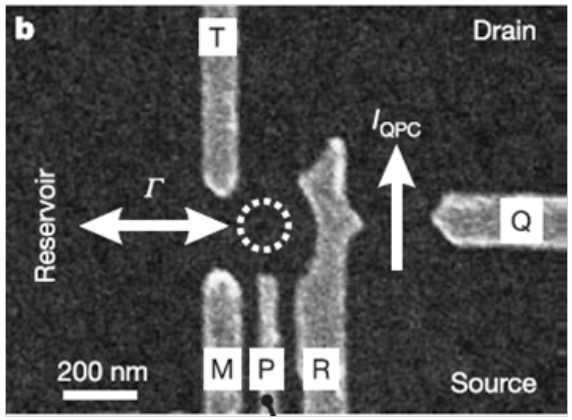
AGH, Faculty of Physics and Applied Computer Science

Presentation plan

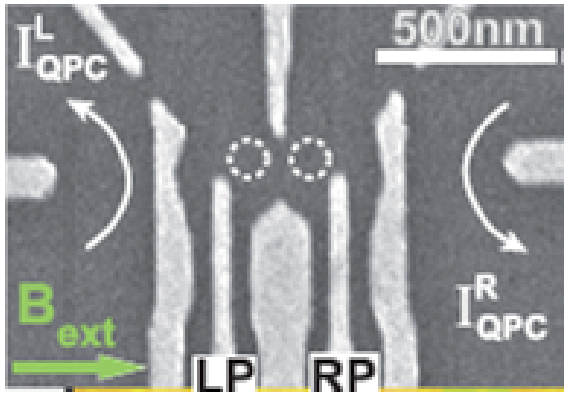
1. Quantum computer qubit
2. Spin initialization - How is it done?
3. The proposed nanodevice
4. How we do it? - The Rashba spin-orbit interaction
5. Brief summary

Quantum computer qubit

One of the many possible quantum computer implementations is based on the spin of an electron trapped in a semiconductor nananostructure.

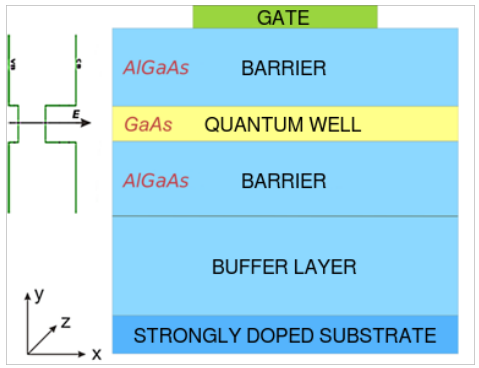


(a) Elzerman et al., Science 333, 1269 (2011)



(b) Nowack, Nature 430, p. 431–435 (2004)

Advantages: easily integrable with classical electronics, scalability.



Quantum computer qubit

The concept of a qubit based on the spin of an electron trapped in semiconductor quantum dots first appeared in a (20 years old) paper by Loss and DiVincenzo:

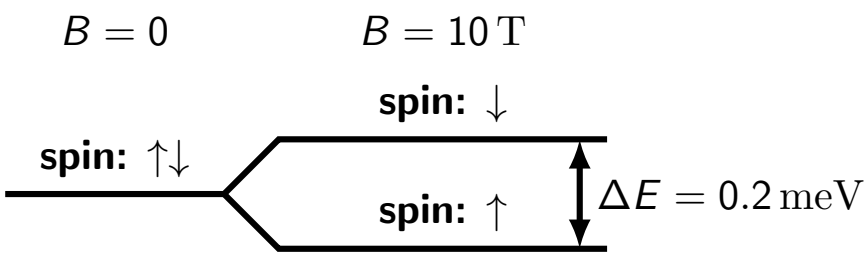
Loss, DiVincenzo, Phys. Rev. A **57**, 120 (1998)

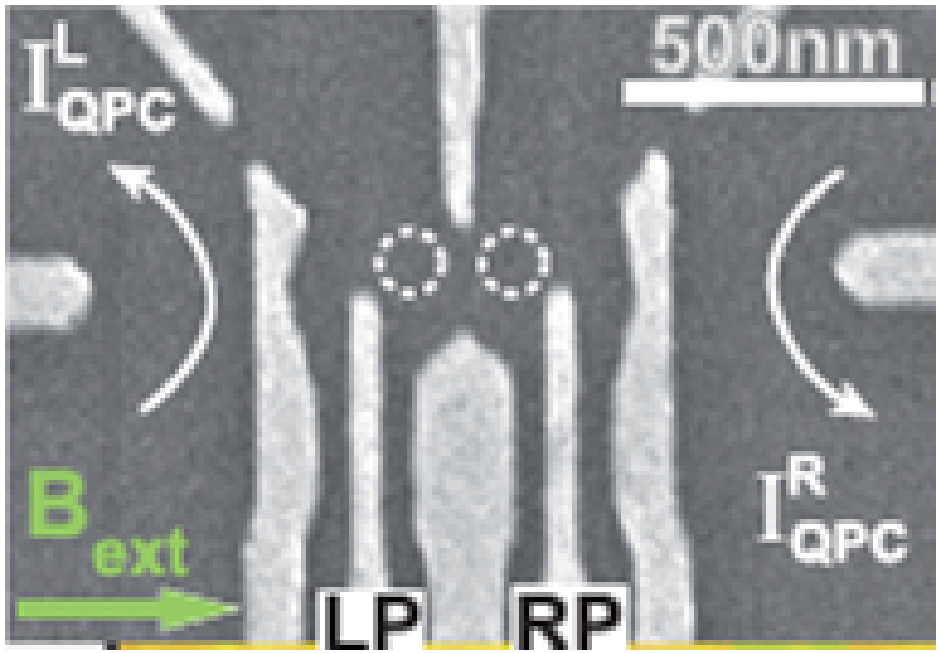
Question: Why don't we have a working quantum processor?
Answer: Due to the lack of a fast and effective spin initialization method.

A fast quantum interface between spin qubits of different codes,
A. Noiri et al., arXiv:1804.04764 (2018).

Initialization in a strong magnetic field

The only currently available electron spin initialization method (in described systems) is based on splitting the spin levels in a strong magnetic field.



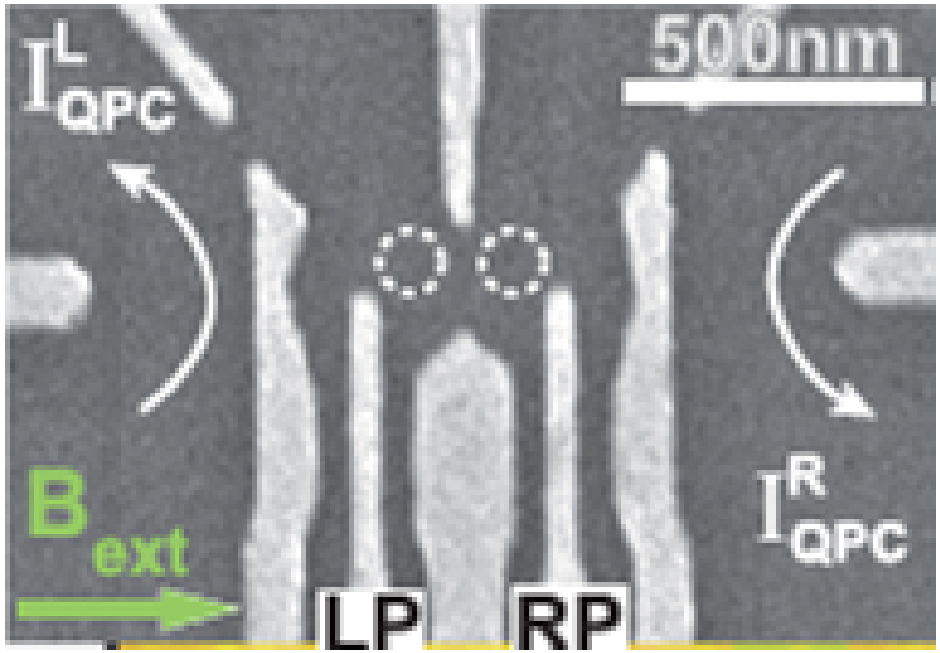


We apply the magnetic field and wait until the electron relaxes to the ground state. The method works but it is slow and requires very low temperatures ($10\,\mathrm{mK}$).

Initialization in a strong magnetic field

In one of experiments done at the University of Delft (Holand) all necessary qubit operations were achieved.

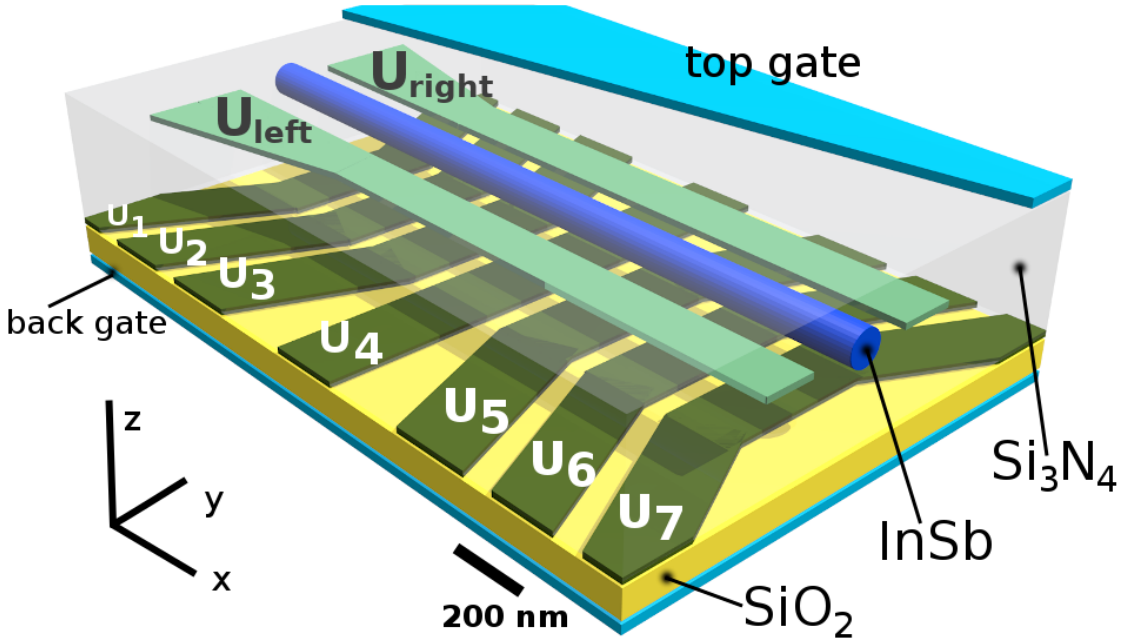
K. C. Nowack et al., Science **333**, 1261 (2011).



Spin initialization with fidelity of the order of 99% takes a couple of ns. The estimated spin coherence time is only about $20\,\mathrm{ns}$. **We need a faster method!**

Proposed nanodevice

Let us create a nanodevice capable of trapping a single electron and performing all the necessary quantum operations on its spin. Setting the spin in a desired direction (initialization) **must be very fast**.

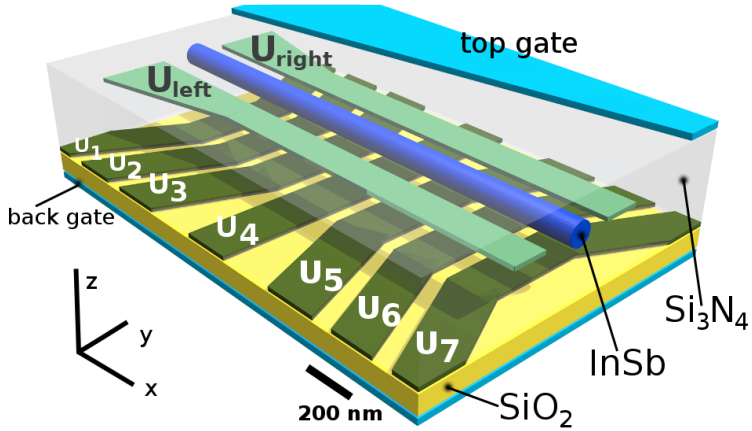


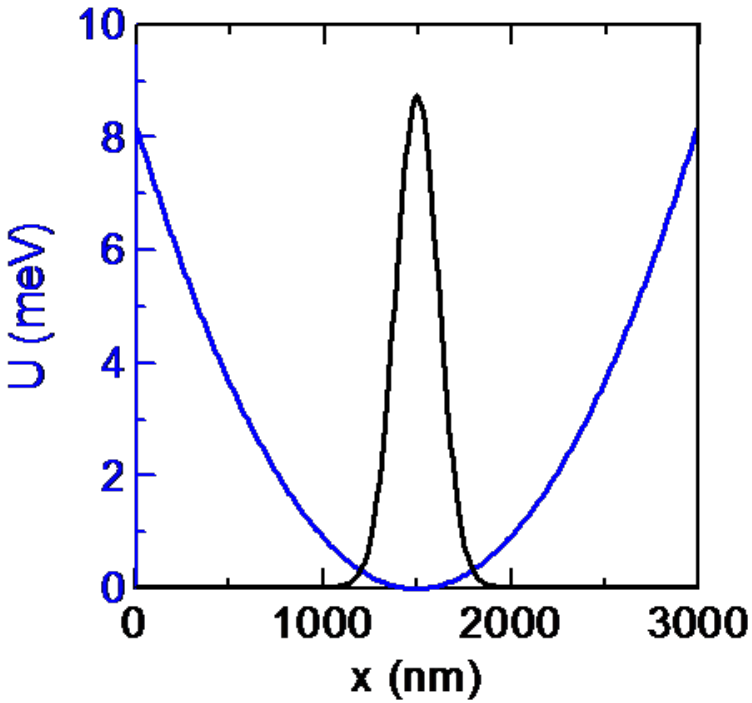
Bednarek, Pawłowski, Górski, Skowron, Phys. Rev. Applied **11**, 034012 (2019)

Proposed nanodevice

1. To the gates $U_{1..7}$ we apply voltages forming an approximately parabolic confinement potential.
2. A single electron is inserted into the wire.
3. The electron relaxes to the ground state.
4. The electron is interacted with by using the Rashba spin-orbit interaction (SOI) and voltages, applied to the gates.

The spin-orbit interaction is a relativistic correction to the Hamiltonian but in most semiconductors it is strong enough to facilitate spin rotations.

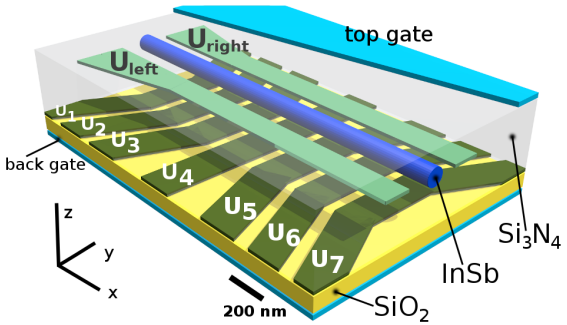




Spin-orbit interaction in a nanowire

In a bulk semiconductor with an electric field **E** present the SOI Hamiltonian is given by

$$\tilde{\mathbf{H}}_{\mathrm{so}} = \frac{\alpha_{\mathrm{so}}|\mathbf{e}|}{\hbar}\,(\mathbf{E}\times\hat{\mathbf{p}})\cdot\hat{\sigma}. \qquad (1)$$



Our nanodevice constricts motion to the x axis, thus the Hamiltonian reduces to

$$\hat{H}_{\mathrm{so}} = \frac{\alpha_{\mathrm{so}}|e|}{\hbar}\,(E_z\sigma_y - E_y\sigma_z)\,\hat{p}_x. \qquad (2)$$

If we apply a strong electric field along the y axis, namely E_y, the second term dominates

$$\hat{H}_{\mathrm{so}} = -\frac{\alpha_{\mathrm{so}}|e|}{\hbar}E_y\sigma_z\hat{p}_x. \qquad (3)$$

Spin-orbit interaction in a nanowire

The full Hamiltonian assumes the form

$$\hat{H} = \frac{\hat{p}_x^2}{2m^*} + U(x) + \hat{H}_{\mathrm{so}}. \tag{4}$$

If we assume a **parabolic confinement potential** and **apply an electric field** E_y we obtain

$$\hat{H} = \frac{\hat{p}_x^2}{2m^*} + \frac{1}{2} m^* \omega^2 \hat{x}^2 - \frac{\alpha_{\mathrm{so}} |e|}{\hbar} E_y \sigma_z \hat{p}_x. \tag{5}$$

The eigenstates of this hamiltonian can be easily found in the momentum representation.

Spin-orbit interaction in a nanowire

Eigenvalue equation:

$$\left[\frac{\hat{p}_x^2}{2m^*} + \frac{1}{2}m^*\omega^2\hat{x}^2 - \frac{\alpha_{\mathrm{so}}|e|}{\hbar}E_y\langle\sigma_z\rangle\hat{p}_x\right]\Psi = E\Psi. \tag{6}$$

The ground state:

$$\Psi = \sqrt{\frac{2\beta}{\pi}}e^{-\beta x^2}e^{iqx} \tag{7}$$

$$\text{where:} \quad \beta = \frac{m^*\omega}{2\hbar} \quad \text{and} \quad q = \frac{m^*\alpha_{\mathrm{so}}|e|E_y}{\hbar^2}\langle\sigma_z\rangle.$$

The Gaussian wavepacket is multiplied by a plane wave, so **it should move**, **but it does not**[1] until we turn off the electric field.

[1] wlewo.exe wprawo.exe

Spin-orbit interaction in a nanowire

We can also induce electron's movement if we proceed in the other way around. First, the electron is relaxed to the ground state and then the electric field E_y is turned on abruptly. **The resulting electron motion is reversed**[2]**.**
If we use a spinor representation, the spin-degenerate ground state is given by:

$$\Psi_{\uparrow} = \sqrt{\frac{2\beta}{\pi}} \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-\beta x^2} e^{iqx} \qquad \Psi_{\downarrow} = \sqrt{\frac{2\beta}{\pi}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{-\beta x^2} e^{-iqx} \tag{8}$$

$$\text{where:} \quad \beta = \frac{m^* \omega}{2\hbar} \quad \text{and} \quad q = \frac{m^* \alpha_{\text{so}} |e| E_y}{\hbar^2}.$$

Given this representation we can construct a wavefunction of any possible spin orientation (e.g. equally-weighted linear combination).

[2] spinup.exe spindown.exe

Abrupt turning-on the SOI

- No initial electric field ($E_y = 0$).
- The initial state with spin oriented along the x axis given by

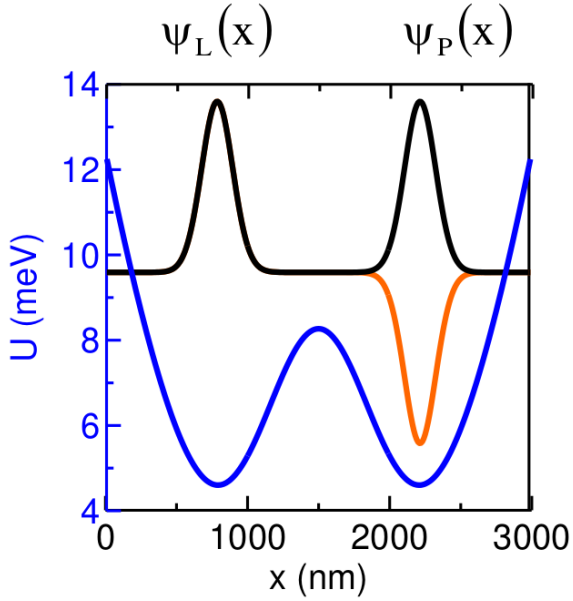
$$\mathbf{\Psi_x} = \frac{1}{\sqrt{2}} \left(\mathbf{\Psi}_{\uparrow} + \mathbf{\Psi}_{\downarrow} \right) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-\beta x^2}. \tag{9}$$

- After the E_y field is turned on, the spin-up and spin-down components behave differently[3].

[3] separacja.exe

Second qubit

After the spin separation we obtain an interesting state. We can define the, so-called **charge qubit** which assigns qubit states to the presence of an electron in the left or in the right half of the nanodevice. Let us denote these states as:



$$|L\rangle = \Psi_L(x) \qquad |R\rangle = \Psi_R(x). \tag{10}$$

The presented state can be written down as

$$\mathbf{\Psi_K} = \begin{pmatrix} \Psi_L(x) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ \Psi_R(x) \end{pmatrix} = |\uparrow\rangle\,|L\rangle + |\downarrow\rangle\,|R\rangle. \tag{11}$$

Schrödinger-cat state

$$|\uparrow\rangle\,|L\rangle + |\downarrow\rangle\,|R\rangle$$

A similar state was obtained experimentally by D. Wineland in ion traps[4].

The only difference: spin states replaced by two **internal (electronic) states of the trapped ion**.

Wineland obtained the so-called **Schrödinger's cat states**

$$|\uparrow\rangle\,|\text{☺}\rangle + |\downarrow\rangle\,|\text{☹}\rangle$$

J. Pawłowski, M. Górski, G. Skowron, and S. Bednarek Phys. Rev. B **96**, 115308 (2017)

[4] Monroe, Meekhof, King, Wineland, Science **272**, 1131 (1996)

Entangled states

Our state is an example of **entangled states**[5].

$$|\uparrow\rangle\,|L\rangle + |\downarrow\rangle\,|R\rangle$$

A two-qubit state is called entangled if it is not possible to express it as a product of two one-qubit states.

Example 1:

$$|\uparrow\rangle\,|L\rangle + |\downarrow\rangle\,|L\rangle + |\uparrow\rangle\,|R\rangle + |\downarrow\rangle\,|R\rangle$$

is not entangled because it can be expressed as:

$$(|\uparrow\rangle + |\downarrow\rangle)(|L\rangle + |R\rangle).$$

Example 2 (our **entangled** *state):*

$$|\uparrow\rangle\,|L\rangle + \cancel{|\downarrow\rangle\,|L\rangle} + \cancel{|\uparrow\rangle\,|R\rangle} + |\downarrow\rangle\,|R\rangle.$$

The state of the whole system is better defined than its components:

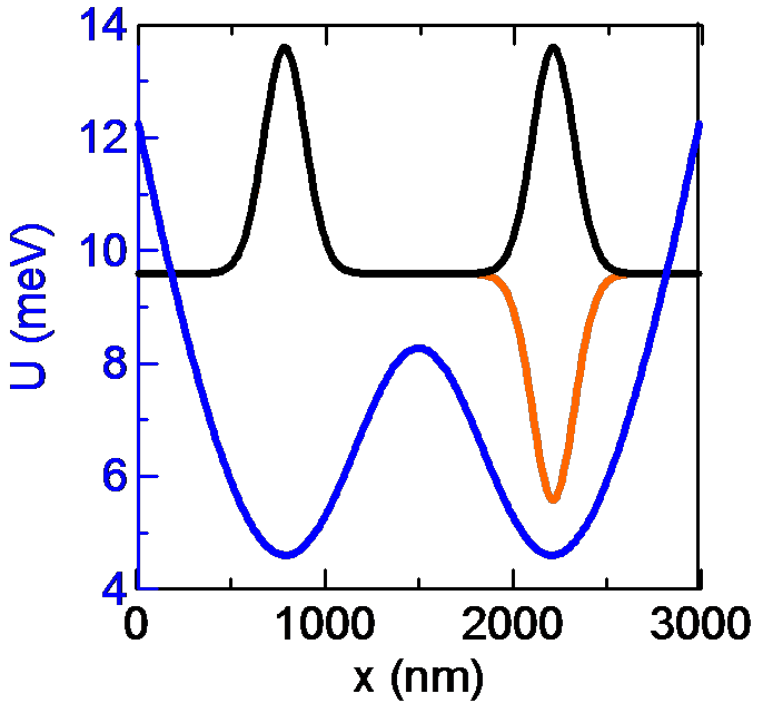
$$|\uparrow\rangle + |\downarrow\rangle \qquad \text{and} \qquad |L\rangle + |R\rangle.$$

[5] E. Schrödinger, Naturwissenschaften **23**, 807 (1935)

What can we do with our Schrödinger-cat state?

Spin separation has already been done:

- Left dot $\Longrightarrow$ Spin-up
- Right dot $\Longrightarrow$ Spin-down



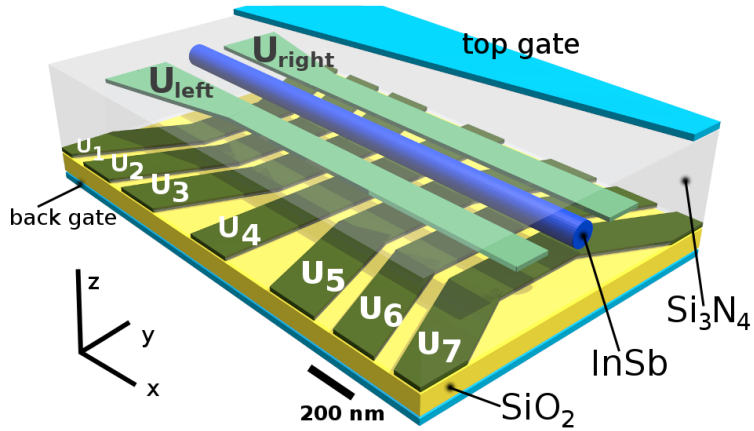
We can do more:
If we rotate spin in the right dot by an angle $\varphi = \pi$ while keeping spin in the left dot unchanged we **arrive at spin oriented upwards in the entire nanodevice**.

Spin rotation in the right dot

Let us see what SOI we can obtain:

$$\hat{H}_{\mathrm{so}} = \frac{\alpha_{\mathrm{so}}|e|}{\hbar}\left(E_z\sigma_y - E_y\sigma_z\right)\hat{p}_x. \qquad (12)$$

So far the SOI was generated using lateral gates that create the electric field E_y.

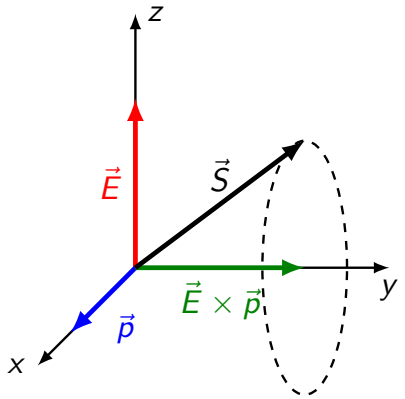


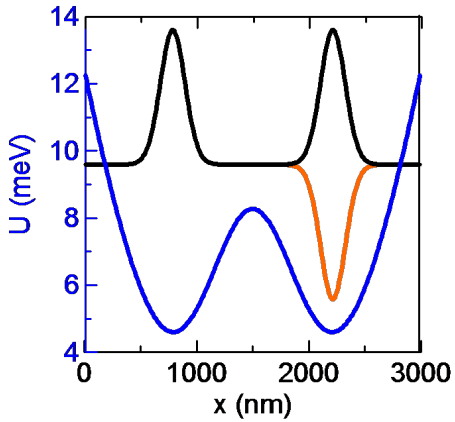
We can also create the electric field E_z by applying a voltage across the substrate and the top gate. In such a case, the SOI is described by

$$\hat{H}_{\mathrm{so}} = \frac{\alpha_{\mathrm{so}}|e|}{\hbar}E_z\sigma_y\hat{p}_x. \qquad (13)$$

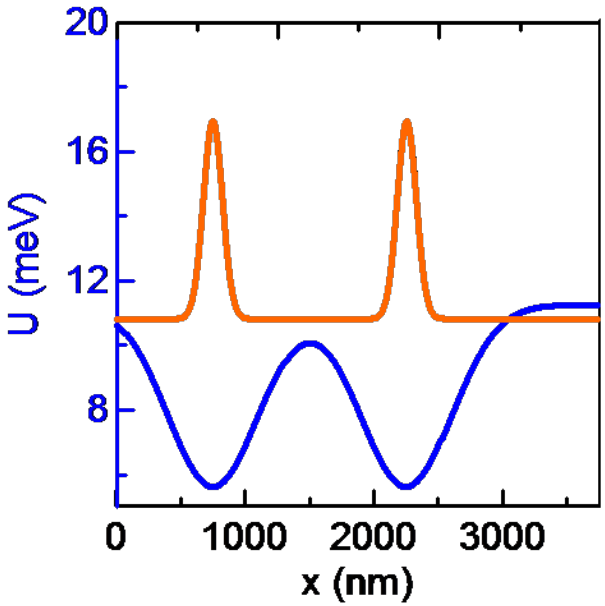
Spin rotation in the right dot

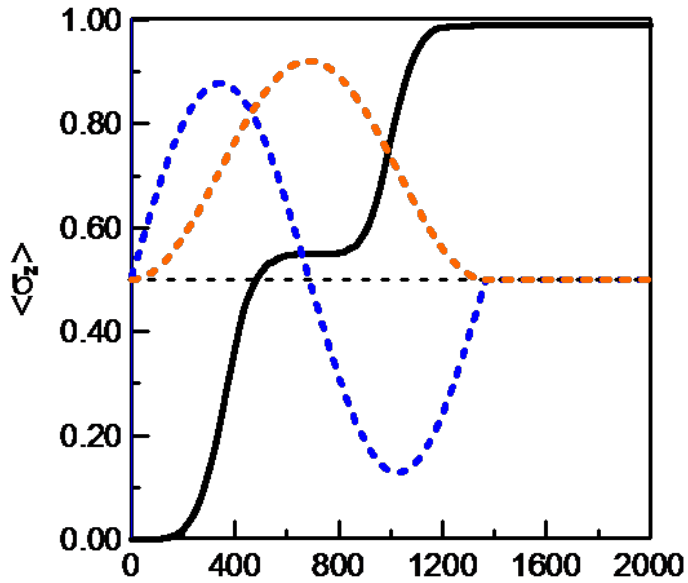
$$\hat{H}_{\mathrm{so}} = \frac{\alpha_{\mathrm{so}}|e|}{\hbar} E_z \sigma_y \hat{p}_x$$





If we force motion of the electron along the x axis with presence of an electric field E_z we can induce rotation of spin about the y axis[6].



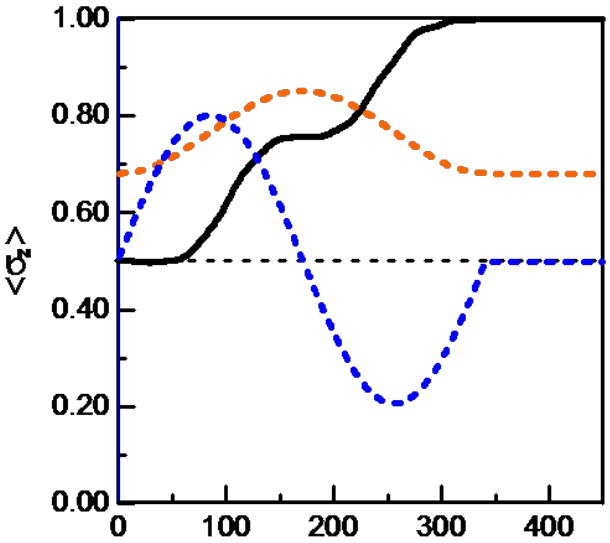


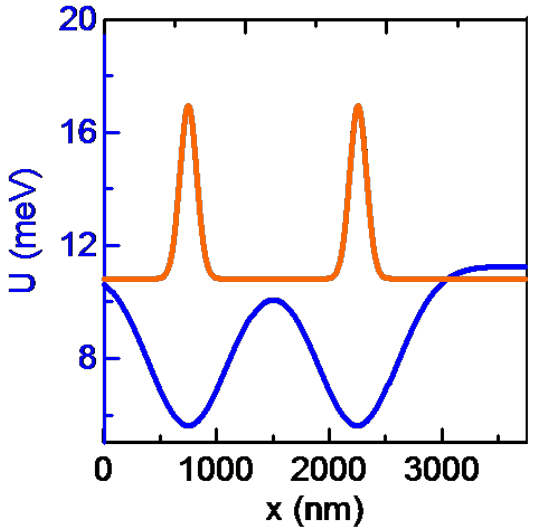
[6] obrot.exe

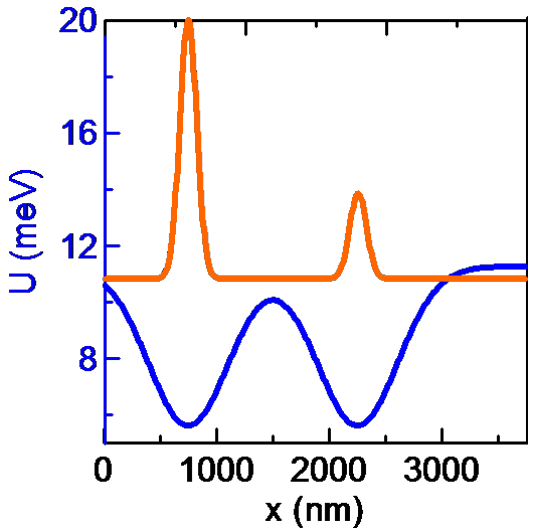
Spin rotation in the right dot

So far, the initial spin was in parallel to the x axis, thus it was an equally weighted linear combination of spin up and down along the z axis. Now, let us assume a **non-equally weighted linear combination**[7]:

$$\chi = \frac{\sqrt{3}}{2}\left|\uparrow\right\rangle + \frac{1}{2}\left|\downarrow\right\rangle$$



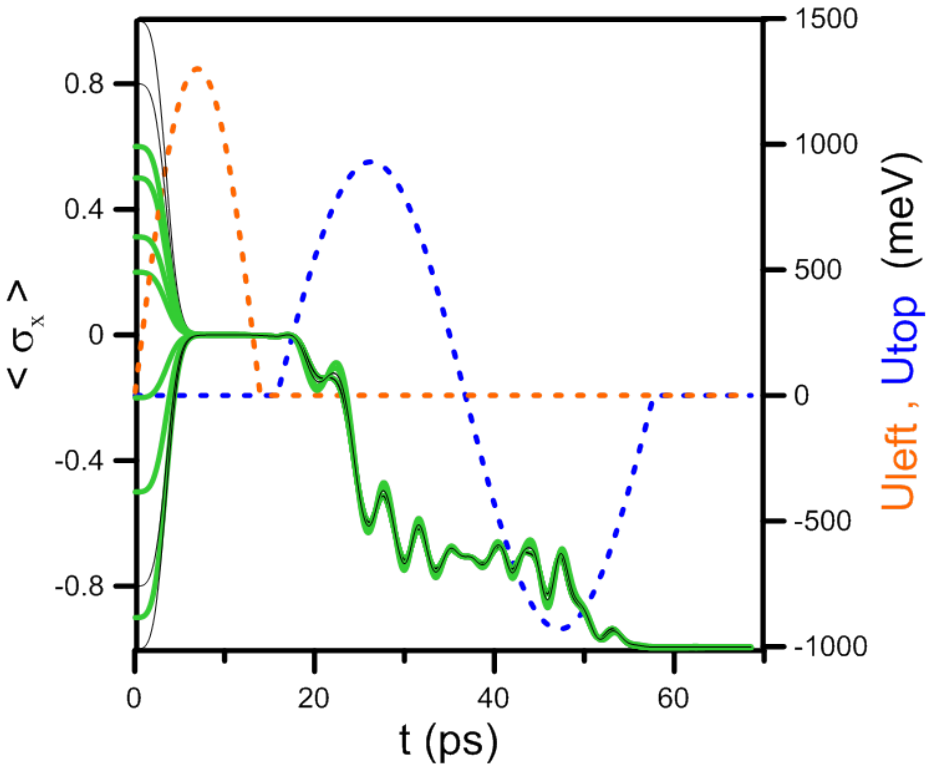




[7] inny.exe obrot1.exe

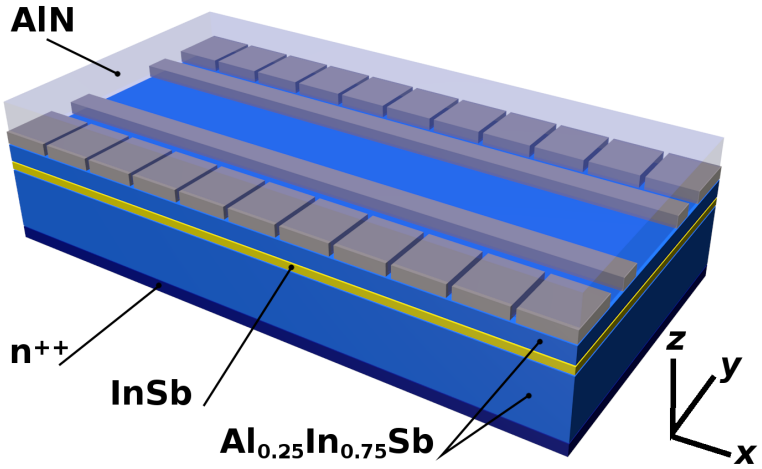
Complete spin initialization procedure

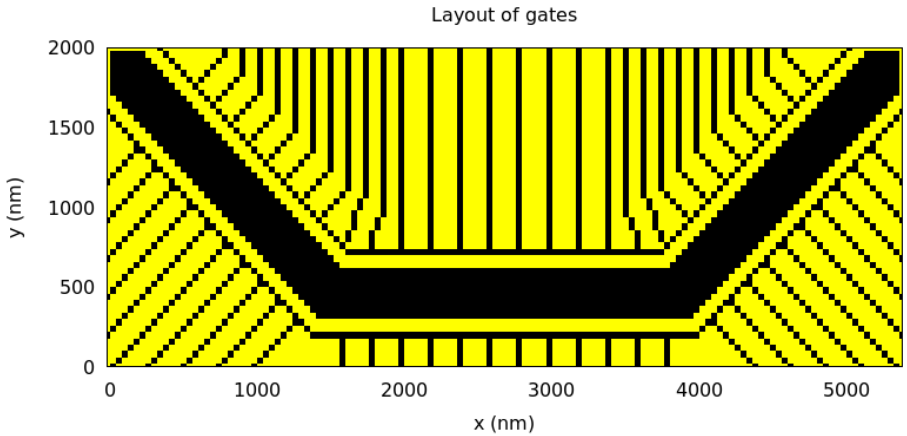
Initialization can be performed faster if we rotate spins in **both halves** of the nanodevice (not just one) **in antiphase**. The resulting spin will be oriented along the x axis.



- Regardless of the initial spin orientation at the end of the procedure the spin is oriented along the x axis.
- **The initialization time does not exceed 60 ps, which is significantly less than the spin coherence time (about 20 ns)**

Yet another nanodevices

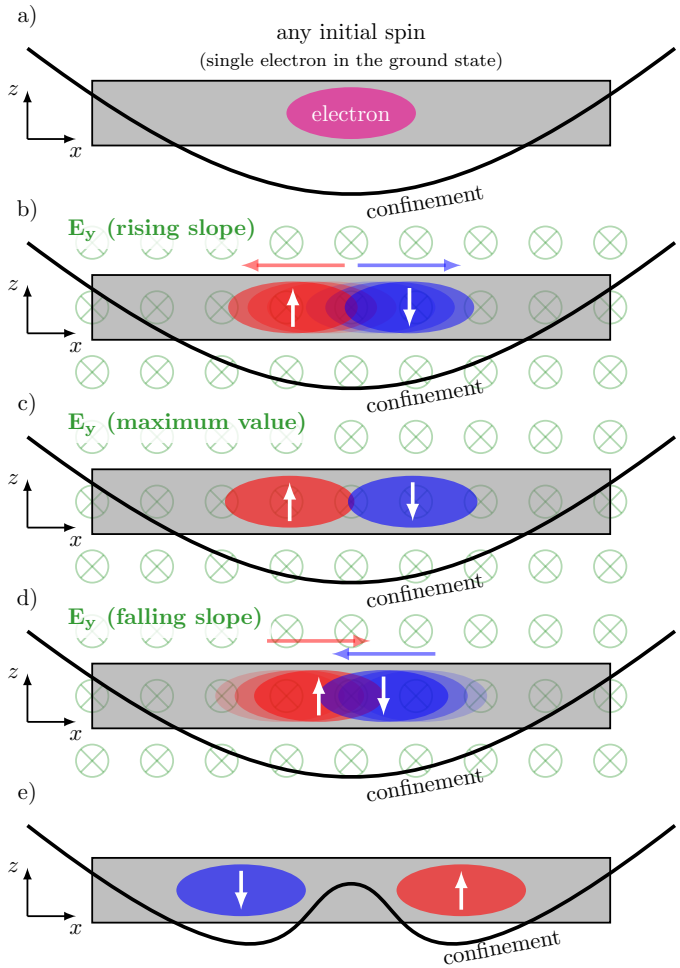




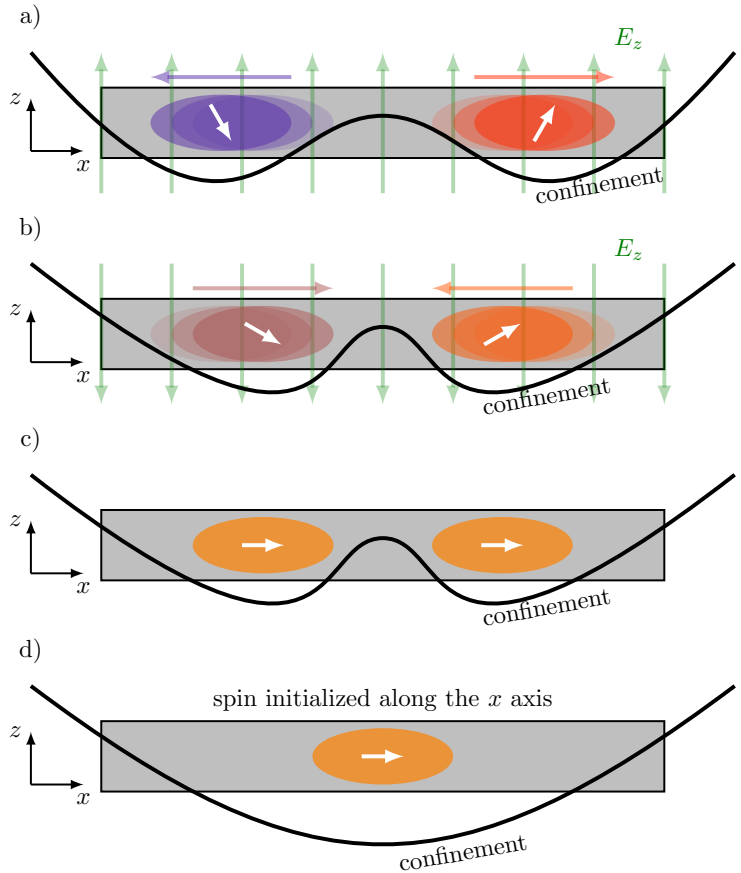
- All employ the Rashba spin-orbit interaction for spin initialization;
- planar nanostructures instead of a quantum wire;
- slightly different procedure due to technological limitations;
- slower initialization time;
- better integration with existing electronics and scalability.

To sum everything up...

Stage 1
(spin separation)



Stage 2
(spin rotation)



Computational methods

The time evolution of electron's wavefunction was calculated by solving the time-dependent Schrödinger equation

$$i\hbar\frac{\partial}{\partial t}\mathbf{\Psi}(x,t) = \hat{H}(x,t)\mathbf{\Psi}(x,t)$$

$$\hat{H}(x,t) = -\frac{\hbar^2}{2m^*}\frac{\partial^2}{\partial x^2} + U(x,t) + \hat{H}_{\mathrm{so}}(x,t)$$

The potential in the nanodevice was calculated using the Poisson equation

$$\nabla^2\varphi(\mathbf{r},t) = \frac{\rho(\mathbf{r},t)}{\varepsilon_0\varepsilon} \qquad \Longrightarrow \qquad U(\mathbf{r},t) = -|e|\varphi(\mathbf{r},t)$$

Due to the time-varying voltages applied to the gates, the Poisson equation is solved for every time-step.

Summary

- Simulations were performed for a concrete (described) realistic nanodevice design.
- They take into account all important material parameters.
- The computational methods were tested with nanodevices for which experimental data were available and the results could be compared.
- Good consistency of our results with experimental data allows us to believe that our nanodevices will work according to our predictions.
- Our nanodevice design allows for an ultrafast spin initialization.

Conclusion: It is high time the experimental research in this area was resumed.

Authors' literature:

- S. Bednarek et al., *Ultrafast Spin Initialization in a Gated InSb Nanowire Quantum Dot*, Phys. Rev. Applied **11**, 034012 (2019)
- S. Bednarek et al., *All-electric single electron spin initialization*, *New J. Phys.* **19** 123006 (2017)
- Pawłowski et al., Phys. Rev. B **96**, 115308 (2017)