Can you win a quantum game ?

Katarzyna Rycerz

Institute of Computer Science, AGH

kzajac@agh.edu.pl

3 April 2019

Overview

1. Non-cooperative Game Theory: basics and examples
2. Quantum Game
   - Rules and definitions
   - Examples: Penny Flip and Prisoners Dilemma
   - Eisert-Wilkens-Lewenstein (EWL) Scheme
3. Games with unawareness
4. Experimental Realisations
5. Quantum Bayesian Networks approach
6. Quantum Games - applications
7. Summary

Non-cooperative Game Theory

Motto (Su Tsu, The Art of War, 500 BCE, China)

Knowing the other and knowing oneself, In one hundred battle no danger,
Not knowing the other and knowing oneself, One victory for one loss,
Not knowing the other and not knowing oneself, In every battle certain defeat

The mathematical fundation of:

- making optimal decisions (i.e. strategic choices)
- in competitive situation (i.e. game)
- based on available information

When a strategy is optimal ?

Nash Equilibrium

No player can benefit by changing his strategy while the other players keep theirs unchanged.
More formally, if σ^* is a strategy profile being Nash equilibrium, then for every player i and every strategy σ_i, the payoff function for player i satisfies: $u_i(\sigma^*) \geq u_i(\sigma_i, \sigma^*_{-i})$

Pareto Optimal

Outcome cannot be improved upon without hurting at least one player.
More formally, strategy profile σ^* is Pareto optimal, if there are no other profile strategy σ' that for payoff function u_i for each player i $u_i(\sigma') \geq u_i(\sigma_i^*)$ with $u_i(\sigma') \geq u_i(\sigma_i^*)$ for some i

Example - Prisoners Dillema

- two prisoners (players)
- two strategies:
  - C - cooperate with the other prisoner (remain silent)
  - D - defeat (testifying that the other committed the crime)
- payoffs $T > R > P > S$, $2R > S + T$

		Prisoner B	
		Cooperate	**Defeat**
Prisoner A	**Cooperate**	(R,R)	(S,T)
	Defeat	(T,S)	(P,P)

Table: Strategic form of Prisoners Dillemma

Pareto Optimal

		Prisoner B	
		Cooperate	**Defeat**
Prisoner A	**Cooperate**	(3,3)	(0,5)
	Defeat	(5,0)	(1,1)

Table: Pareto Optimal in Prisoners Dillemma

Looking for Nash equilibrium

		Prisoner B	
		Cooperate	**Defeat**
Prisoner A	**Cooperate**	$\downarrow$ (3,3) $\rightarrow$	(0,5)
	Defeat	(5,0)	(1,1)

Table: Looking for Nash equilibrium for Prisoners Dillemma

Looking for Nash equilibrium

		Prisoner B	
		Cooperate	**Defeat**
Prisoner	**Cooperate**	(3,3)	$\downarrow$ (0,5)
A	**Defeat**	(5,0) $\rightarrow$	(1,1)

Table: Looking for Nash equilibrium for Prisoners Dillemma

Looking for Nash equilibrium

		Prisoner B	
		Cooperate	**Defeat**
Prisoner A	**Cooperate**	(3,3)	(0,5)
	Defeat	(5,0)	(1,1)

Table: Nash equilibrium for Prisoners Dillemma

Quantum Game Rules

A quantum game:

- uses concepts from quantum computing
- can be reduced to its classical counterpart
- shows something more that its classical counterpart

Example Definition

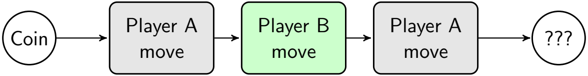
a pure strategy quantum game is a unitary function:
$Q : \otimes_{i=1}^{n} \mathbb{C}P^{d_i} \to \otimes_{i=1}^{n} \mathbb{C}P^{d_i}$ where $\mathbb{C}P^{d_i}$ is a d_i-dimensional complex projective Hilbert space, as well as a set of outcomes with a notion of non-identical preferences defined over its elements, one per player.

Reference

Khan Faisal, Solmeyer Neal, Balu Radhakrishnan and Humble, Travis. (2018). Quantum games: a review of the history, current state, and interpretation. Quantum Information Processing, Volume 17, Issue 11, article id. 309, 42 pp.

Classical Penny Flip

- two players, one coin, no one sees intermediate states of the coin, three steps:
  1. first player chooses to flip(F) or not to flip (N) the coin
  2. second player chooses to flip(F) or not to flip (N) the coin
  3. first player chooses to flip(F) or not to flip (N) the coin



		Player 2	
		N	**F**
Player 1	**NN**	(1,-1)	(-1,1)
	NF	(-1,1)	(1,-1)
	FN	(-1,1)	(1,-1)
	FF	(1,-1)	(-1,1)

Table: Strategic form of Penny Flip

Quantum Penny Flip

- state of the coin = state of the qbit
- action on the coin = any unitary gate (quatum strategy)
- not flipping the coin = identity matrix
- flipping the coin = NOT (σ_x or X) matrix

$$|0\rangle \; - \boxed{U_1} - \boxed{U_2} - \boxed{U_3} \tag{1}$$

Reference

D. A. Meyer, Quantum strategies, Phys. Rev. Lett., vol. 82, pp. 10521055, Feb 1999.

Quantum Penny Flip – Results

- if only player 1 is allowed to use quantum stategies he/she always wins
- if both players are allowed to use pure quantum stategies in zero sum game = Nash equilibrium need no exists
- if both players are allowed to choose quantum strategies with classical probability (mixed quantum stategies)= Nash equilibrium would always exists

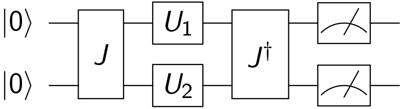
$$|0\rangle \; - \boxed{H} - \boxed{P_2} - \boxed{H}$$

$$P_2 \in \{I, X\}$$

$$|0\rangle \xrightarrow[\text{player 1}]{\text{H}} \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \xrightarrow[\text{player 2}]{\text{I or X}} \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \xrightarrow[\text{player 1}]{\text{H}} |0\rangle$$

Eisert-Wilkens-Lewenstein (EWL) Scheme

- scheme applied to Prisoners Dillema
- prisoners strategies = unitary gates (quatum strategies)
- starts always from $|00\rangle$
- cooperation = identity matrix
- defeat = NOT (σ_x or X) matrix
- payoff is based on results of the final measurement ($|0\rangle$ acts as cooperation, $|1\rangle$ acts as defeat)



$$J = \frac{1}{\sqrt{2}}(\mathbb{1} + i\sigma_x)$$

Reference

J. Eisert, M. Wilkens, and M. Lewenstein, Quantum games and quantum strategies, Phys. Rev. Lett., vol. 83, pp. 30773080, Oct 1999

EWL Scheme - results

- in general, a prisoner strategy can be any unitary matrix represented by three parameters e.g. $U(\theta, \phi, \alpha) = \begin{pmatrix} e^{-i\phi} \cos \frac{\theta}{2} & e^{i\alpha} \sin \frac{\theta}{2} \\ -e^{-i\alpha} \sin \frac{\theta}{2} & e^{i\phi} \cos \frac{\theta}{2} \end{pmatrix}, \theta \in [0, \pi], \alpha, \phi \in [-\pi, \pi]$.
- but original work restricts the strategic space to the 2-parameter set: $U(\theta, \alpha) = \begin{pmatrix} e^{i\alpha} \cos \frac{\theta}{2} & \sin \frac{\theta}{2} \\ - \sin \frac{\theta}{2} & e^{-i\alpha} \cos \frac{\theta}{2} \end{pmatrix}, \theta \in [0, \pi], \alpha \in [0, \frac{\pi}{2}]$
- this restriction allows to find Nash equilibrium that is also Pareto optimal
- but in general – for any strategy $U(\theta, \phi, \alpha)$ there exists answer $U(\theta + \pi, \alpha, \phi - \frac{\pi}{2})$ that changes result (defeats,cooperates) into (cooperates,defeats) $=$ no Nash equilibrium !

Games with unawareness

- formalism that describes situation when players have different knowledge about the game
- V is set of different player views :
  - $v = 0$ Game designer view
  - $v = 1$ Alice thinks ...
  - $v = 2$ Bob thinks ...
  - $v = 12$ Alice thinks that Bob thinks ...
  - $v = 121$ Alice thinks that Bob thinks that Alice thinks ...
- $\{G_v\}_{v \in V}$ – family of games regarding different views
- extended Nash equilibrium
- designed for classical games, extended to quantum games

Reference

Y.Feinberg, Games with Unawareness Working Paper No. 2122, Stanford Graduate School of Business (2012).

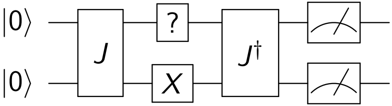
P. Frackiewicz , Quantum Penny Flip Game with Unawareness Quantum Information Processing (2019)

Prisoner Dilemma with unawareness

- Alice knows that they play quantum (Q) game
- Bob thinks that they play classical (C) game

$$\{G_v\}_1 = \begin{cases} \text{(Q,Q)} & \text{v=0} \\ \text{(Q,C)} & \text{v=1} \\ \text{(C,C)} & \text{v=2} \\ \text{(C,C)} & \text{v=12} \\ \text{(C,C)} & \text{v=21} \end{cases}$$

Bob will defeat, what is Alice best move?

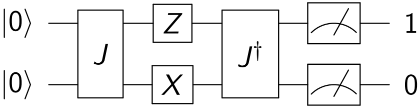


Prisoner Dilemma with unawareness

- Alice knows that they play quantum (Q) game
- Bob thinks that they play classical (C) game

$$\{G_v\}_1 = \begin{cases} (\mathrm{Q},\mathrm{Q}) & \mathrm{v}=0 \\ (\mathrm{Q},\mathrm{C}) & \mathrm{v}=1 \\ (\mathrm{C},\mathrm{C}) & \mathrm{v}=2 \\ (\mathrm{C},\mathrm{C}) & \mathrm{v}=12 \\ (\mathrm{C},\mathrm{C}) & \mathrm{v}=21 \end{cases}$$

Bob will defeat (σ_x), Alce best move is σ_z

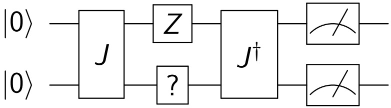


Prisoner Dilemma with unawareness

- Alice knows that they play quantum (Q) game
- Bob knows that Alice plays quatum (Q) game and he playes classical (C) game
- Alice thinks that Bob thinks they play classical (C) game

$$\{G_v\}_2 = \begin{cases} (Q,Q) & v=0 \\ (Q,C) & v=1 \\ (Q,C) & v=2 \\ (C,C) & v=12 \\ (Q,C) & v=21 \\ (C,C) & v=121 \\ (C,C) & v=212 \end{cases}$$

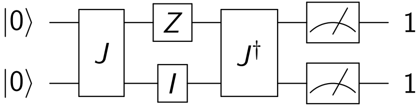
Alice uses σ_Z what is Bob best move?



- Bob knows that Alice plays quantum (Q) game and he playes classical (C) game
- Alice thinks that Bob thinks they play classical (C) game

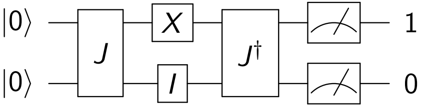
$$\{G_v\}_2 = \begin{cases} (\mathrm{Q,Q}) & \mathrm{v}=0 \\ (\mathrm{Q,C}) & \mathrm{v}=1 \\ (\mathrm{Q,C}) & \mathrm{v}=2 \\ (\mathrm{C,C}) & \mathrm{v}=12 \\ (\mathrm{Q,C}) & \mathrm{v}=21 \\ (\mathrm{C,C}) & \mathrm{v}=121 \\ (\mathrm{C,C}) & \mathrm{v}=212 \end{cases}$$

Alice uses σ_Z and Bob does nothing (I) !

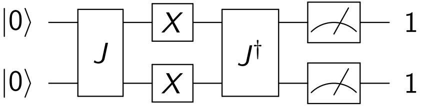


On a higher levels of unawareness ...

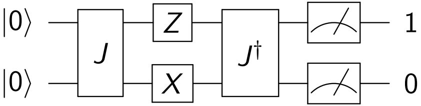
Alice replies



Bob replies



Alice replies



Extended Nash Equilibrium

Family of strategy profiles $\{\sigma_v\}$ for views $v \in V_0$.
Example for $\{G_v\}_2$

$$\sigma_v = \begin{cases} (\sigma_z, I) & \text{v=0} \\ (\sigma_z, \sigma_x) & \text{v=1} \\ (\sigma_z, I) & \text{v=2} \\ (\sigma_x, \sigma_x) & \text{v=12} \\ (\sigma_z, \sigma_x) & \text{v=21} \\ (\sigma_x, \sigma_x) & \text{v=121} \\ (\sigma_x, \sigma_x) & \text{v=212} \end{cases} \tag{2}$$

How to get extended Nash Equilibrium ?

- for $v = 0$ $\sigma_0 = ((\sigma_{Alice})_1, (\sigma_{Bob})_2)$
- for $v = 1, 2, 12, ..$ we get optimal strategies "backwards"
- assumption: my opponent plays the same strategy in the game I consider and in the game I think he considers. $(\sigma_v)_w = (\sigma_v)_{wv}$
- analysis for Alice (v=1...):

Alice		Bob	View	Game
$(\sigma_z$	$\xrightarrow[\text{best answer to}]{\hspace{4em}}$	$\sigma_x)$	v=1	(Q,C)
		the $\uparrow$ *same as*		
$(\sigma_x$	$\xleftarrow[\text{best answer to}]{\hspace{4em}}$	$\sigma_x)$	v=12	(C, C)
the $\uparrow$ *same as*				
$(\sigma_x$		$\sigma_x)$	v=121,...	(C,C)

How to get extended Nash Equilibrium ?

- analysis for Bob (v=2...):

Alice		Bob	View	Game
(σ_z	$\xleftarrow{\text{best answer to}}$	I)	v=2	(Q,C)
the $\uparrow$ *same as*				
(σ_z	$\xrightarrow{\text{best answer to}}$	σ_x)	v=21	(Q, C)
		the $\uparrow$ *same as*		
(σ_x		σ_x)	v=212,...	(C,C)

for $v = 0$

$\sigma_0 = (\sigma_z, I)$

General Method of finding the best answer

- Represent possible strategies as parametrized unitary matrices $U_i(\theta_i, \phi_i, \alpha_i)$
- Using definition of payoff function calculate probability of winning the biggest payoff $P(\theta_1, .., \theta_n, \phi_1, ..., \phi_n, \alpha_1, ..., \alpha_n)$
  - this can be done on the piece of paper or by some symbolic calculation (Mathematica)
- analize propability function to find relations between parameters $\theta_1, .., \theta_n, \phi_1, ..., \phi_n, \alpha_1, ..., \alpha_n$
  - in general not trivial step

Experimental realisations of quantum games

Nuclear Magnetic Resonance

D. Jiangfeng, L. Hui, X. Xiaodong, S. Mingjun at al(2002). Experimental Realization of Quantum Games on a Quantum Computer. Physical review letters. 88. 137902. 10.1103/PhysRevLett.88.137902.

Linear Optics

R. Prevedel, A. Stefanov, P. Walther, A. Zeilinger Experimental realization of a quantum game on a one-way quantum computer New J. Phys. 9, 205 (2007)

Ion trapped quantum computer

N. Solmeyer, N.M.Linke, C.Figgatt at al. Demonstration of a Bayesian quantum game on an ion-trap quantum computer, Quantum Science and Technology, 3(4), 045002, 2018

Prisoners Dilemma on IBM-Q

Challanges:

1. to build J gate (done!)
2. to reduce errors (in progress)

Reference

Filip Galas, Quantum games on IBM-Q
MSc thesis (in preparation)
Talk on KQIS (to be announced)

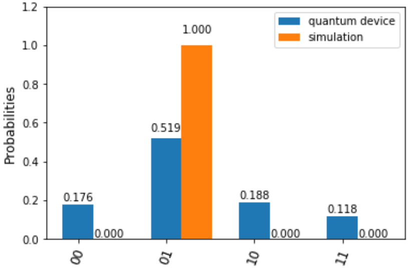


Figure: Comparison of simulation and IBM-Q results for quantum prisoner dillema, when Alice cooperates and Bob defeats.

Quantum Bayesian Networks in Games

1. allow to model decisions that are entangled
2. nodes (or entangled groups of nodes) = density matrices
3. density matrix describes a classical mixture of quantum states.

$$\rho = \sum_{i=1}^{N} p_i \left|\psi_i\right\rangle \left\langle\psi_i\right|$$

4. but can also describe pure (entangled or not) state:

$$\rho = \left|\psi\right\rangle \left\langle\psi\right|$$

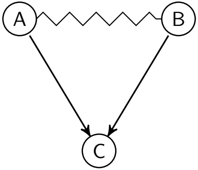


Figure: States of systems A and B are entangled (zigzag line), and there is classic dependence of C on A and B (arrows).

$$\sigma_A \star \sigma_B = \sigma_B^{\frac{1}{2}} \sigma_A \sigma_B^{\frac{1}{2}}$$

$$\rho_{ABC} = \rho_{C|AB} \star (\rho_{AB} \otimes I_C)$$

Quantum Bayesian Networks in Games

- `AcausalNets.jl` library supporting inference in a quantum generalization of Bayesian networks.
- https://github.com/mikegpl/AcausalNets.jl

References

Dariusz Kurzyk, Adam Glos: Quantum inferring acausal structures and themonty hall problem. Quantum Information Processing 15(12), 4927–4937(Dec 2016).
Marcin Przewiezlikowski, Michal Grabowski, Support for high-level quantum Bayesian inference. Engineer Project at ICS AGH, paper accepted at ICCS 2019.
Talk on 22 May on KQIS !

Example application to wireless communication

- base station distributes n-particle entangled quantum state among n individual transmitters, i.e., players,
- players apply local strategies to each part of the quantum state
- after measurement, the players select whether to transmit or wait
- using the quantum resource in this game reduces the probability of transmission collision by a factor of n
- fairness in access management is retained

Reference

O. G. Zabaleta, J. P. Barrang u, and C. M. Arizmendi, Quantum game application to spectrum scarcity problems, Physica A: Statistical Mechanics and its Applications, vol. 466, pp. 455461, 2017

Quantized version of the routing game

- routing games aim at minimizing flow cost through a network
- so called *Nash flow* ensures that all used paths have minimal costs.
- quantum version: a distributed quantum state between players representing the nodes within the network.
- each player is permitted to apply a local quantum strategy (unitary operation)
- finding the minimal network flow cost in terms of latency
- minimal cost is realized with partially entangled state between nodes

Reference

N. Solmeyer, R. Dixon, and R. Balu, Quantum routing games, arXiv preprint arXiv:1709.10500,2017

Other applications ideas

- selection of open access publishing decisions
- quantum description of economics
- quantum error correction or fault tolerant quantum systems require competition between the user and the environment
- SimulaQron simulator for a quantum internet http://www.simulaqron.org/ *"if two players in a network play games, they can use quantum entanglement to coordinate their actions better than what is possible classically and thus win the game more often"*

References

M. Hanauske, S. Bernius, B. Dugall, Quantum game theory and open access publishing, Physica A: Statistical Mechanics and its Applications, Volume 382, Issue 2, 2007, pp 650-664

E.W. Piotrowski, J. Sladkowski, Quantum market games, Physica A: Statistical Mechanics and its Applications, Volume 312, Issues 12, 2002, Pages 208-216

Summary

- a game is quantum if :
  - it extends classical game with any quantum idea
  - can be reduced to its classical counterpart
- several approaches exists, no single standard/definition
- most of the work is kind of theoretical consideration
- some promising work on applying the theory to practical problems, especially in quantum networks