# Automatic Variationally Stable Analysis for Finite Element Computations

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**Abstract**

We introduce the automatic variationally stable finite element (AVS-FE) method [1] for linear scalar-valued convection-diffusion problems. The AVS-FE method uses a first order system integral formulation of the underlying partial differential equations (PDEs) and, in the spirit of the discontinuous Petrov-Galerkin (DPG) method by Demkowicz and Gopalakrishnan [2, 3], employs optimal test functions to ensure discrete stability. The AVS-FE method distinguishes itself by using global $H^1(\Omega)$ and $H(\mathrm{div}, \Omega)$ conforming FE trial spaces and broken Hilbert spaces for the test spaces [1]. Discontinuous function spaces, however, are employed for the test functions and therefore the test functions can be solved locally at the element level by using the DPG philosophy in [2, 3]. Hence, they are optimal in the sense that they guarantee inherently stable FE approximations with best approximation properties in terms of the energy norm. The local contributions of test functions can be numerically solved on each element with high numerical accuracy and do not require the solution of global variational statements. We present 2D numerical results of convection-dominated diffusion problems which show optimal asymptotic convergence rates, and present preliminary results for goal-oriented a posteriori error estimates.

This is a joint work with Albert Romkes (South Dakota School of Mines & Technology, Rapid City, USA) and Victor M. Calo (Curtin University, Perth, Australia).

## References

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