Analysis of Quantum Error Correction Applicability on IBM-Q

Julia Sypień
Supervisor: Katarzyna Rycerz, Ph.D.

Agenda

1. Motivation - quantum computers
2. Introduction to quantum error-correcting codes
3. Quantum error-correction on IBM Q - state of the art
4. A new implementation of the five-qubit quantum error-correcting code and its applicability on IBM Q devices
5. Summary and future work

Motivation

The power of quantum computers

- A qubit, the unit of information in quantum computers can be in a superposition of its base states:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

- Quantum computer can perform computations on multiple states at the same time (!).
- Quantum computers can solve certain existing computational problems in significantly shorter time compared to classical computers.
- Well known example: breaking the RSA (Rivest–Shamir–Adleman) cryptosystem, in polynomial time, using the Shor's factorization algorithm [1].

[1] P. W. Shor, "Polynomial-Time Algorithms for Prime Factorization and Discrete Logarithms on a Quantum Computer," SIAM Journal on Computing. 26. 1484-1509. 10.1137/S0097539795293172, 1997.

IBM Quantum Experience [2]

- Online platform, through which Internet users can access real quantum devices.
- Nine available real quantum devices:
  - one 15-qubits device,
  - seven 5-qubits devices,
  - one 1-qubit device.
- One available quantum simulator, able to simulate up to 32 qubits.

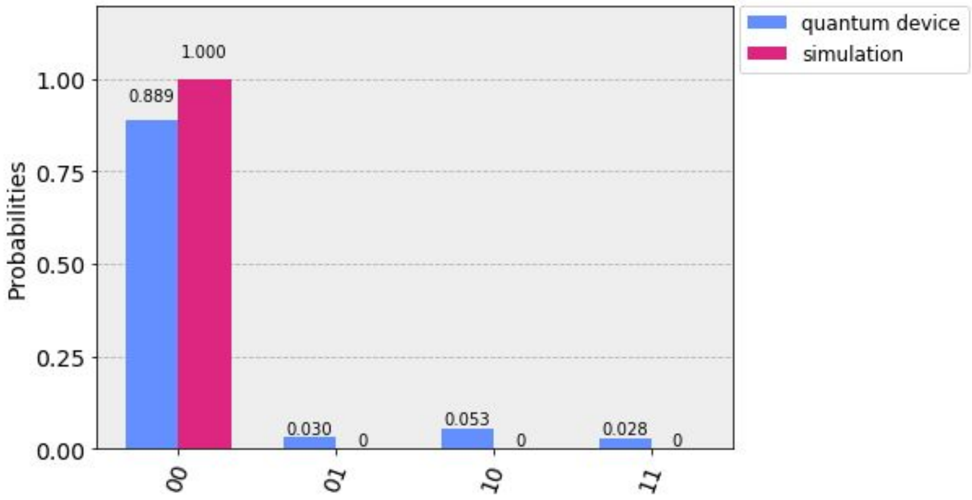
[2] “IBM Q Experience”. https://quantum-computing.ibm.com/

Source: https://teamquest.pl/blog/856_ibm-rommetty-chmura-davos

Limitation of quantum computers - quantum errors

- The state of a qubit in a quantum computer is very fragile and prone to the destructive influence of its outer environment.
- Computations run on quantum computers are highly error-prone
- With high probability, results produced by quantum computers are incorrect.

- Comparison of running the same program on a quantum simulator and on a real quantum device:



Quantum Error Correction

Errors in quantum computers

- Errors acting on quantum states can be thought of as unexpected, random quantum operators applied to these states.
- Any single-qubit quantum operator can be expressed as a linear combination of the Pauli operators (because they form the basis in the quantum operators space):

$$I \equiv \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, X \equiv \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, Y \equiv \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, Z \equiv \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}.$$

Source: https://en.wikipedia.org/wiki/Stabilizer_code

- Any operational quantum error can be expressed as a linear combination of the Pauli operators.

Examples of quantum errors

- Bit-flip error - equivalent of the Pauli X operator acting on a qubit:

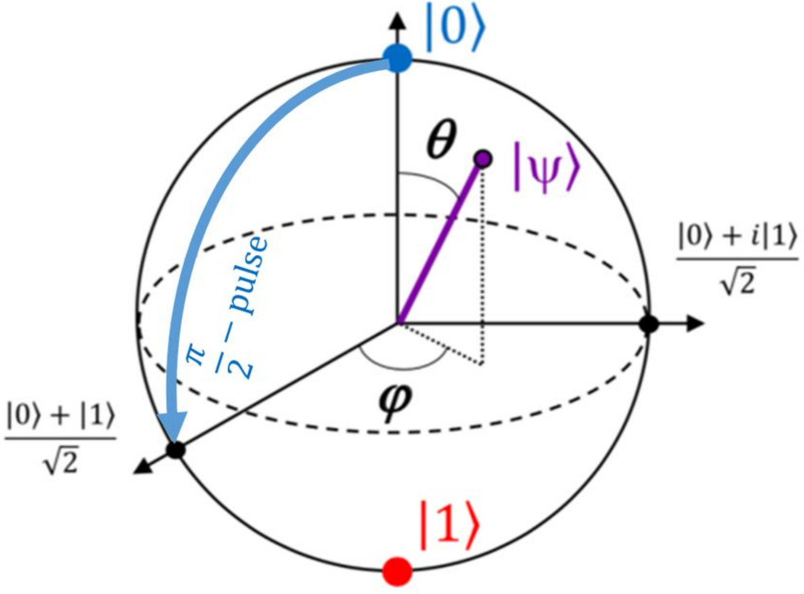
$$
\begin{aligned}
X|0\rangle &= |1\rangle, \\
X|1\rangle &= |0\rangle, \\
|\psi\rangle = \alpha|0\rangle + \beta|1\rangle &\longrightarrow X|\psi\rangle = \beta|0\rangle + \alpha|1\rangle.
\end{aligned}
$$

- Phase-flip error - equivalent of the Pauli Z operator acting on a qubit:

$$
\begin{aligned}
Z|0\rangle &= |0\rangle, \\
Z|1\rangle &= -|1\rangle, \\
|\psi\rangle = \alpha|0\rangle + \beta|1\rangle &\longrightarrow Z|\psi\rangle = \alpha|0\rangle - \beta|1\rangle.
\end{aligned}
$$

Quantum errors - representation on the Bloch Sphere

- A quantum state can be represented as a point on the Bloch Sphere.
- The points on the poles of the sphere are the base states, and any other point represents a superposition of the base states.
- A quantum error can be thought of as an arbitrary rotation of the original state to a different point on the surface of the sphere.



$$|\psi\rangle = cos\frac{\theta}{2}|0\rangle + e^{i\varphi} sin\frac{\theta}{2}|1\rangle$$

Source: https://www.researchgate.net/figure/The-Bloch-sphere-provides-a-useful-means-of-visualizing-the-state-of-a-single-qubit-and_fig1_335028508

Quantum Error Correcting Code (QECC) - notation

- The following notation is used to describe a quantum error-correcting code:

$$[[n, k, d]]$$

where:

- n - the number of qubits representing the original state after encoding
- k - the number of encoded qubits
- d - the code distance (the minimum distance between any two codewords; the minimum number of qubits that have to be changed to obtain a valid codeword from another valid codeword)

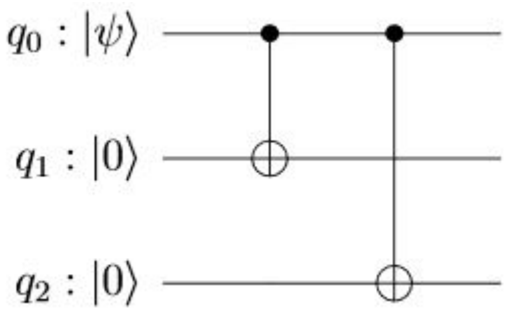
General construction of a quantum error-correcting code

- In general, a qecc consists of the following steps:
    - Encoding
    - Syndrome measurement
    - Error correction
    - Decoding
- “Syndrome measurement” is a procedure where it is checked if an error has occurred, but no measurement is performed in the sense of collapsing a quantum state to one of the base states.

Example: three-qubit bit-flip qecc, encoding

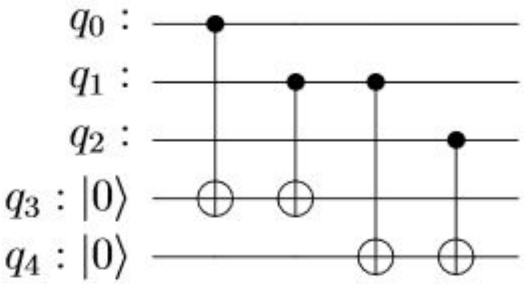
- A simple quantum error-correcting code, able to detect and correct one bit-flip error.
- Encoding:
  - The original state is repeated across three qubits:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \longrightarrow |\psi'\rangle = \alpha|000\rangle + \beta|111\rangle$$



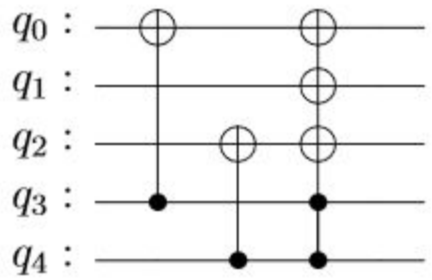
Example: three-qubit bit-flip qecc, syndrome measurement

- Syndrome measurement:
  - By comparing the parity of pairs of qubits it can be deduced which qubit, if any, was flipped.
  - The measured syndrome is stored in two ancilla qubits.



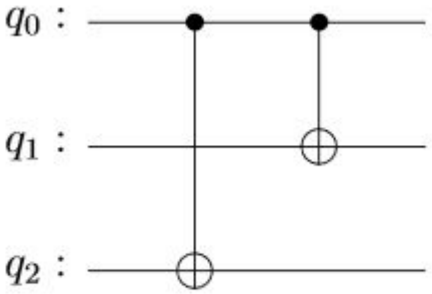
Example: three-qubit bit-flip qecc, error correction

- Error correction:
  - Based on the state of ancilla qubits, the affected qubit is flipped to recover the initial state with the use of controlled gates.



Example: three-qubit bit-flip qecc, decoding

- Decoding:
  - State decoding is performed.



Other examples of quantum error-correcting codes

- Three-qubit phase-flip code
- Nine-qubit Shor’s code
- Five-qubit perfect code
- Seven-qubit Steane code

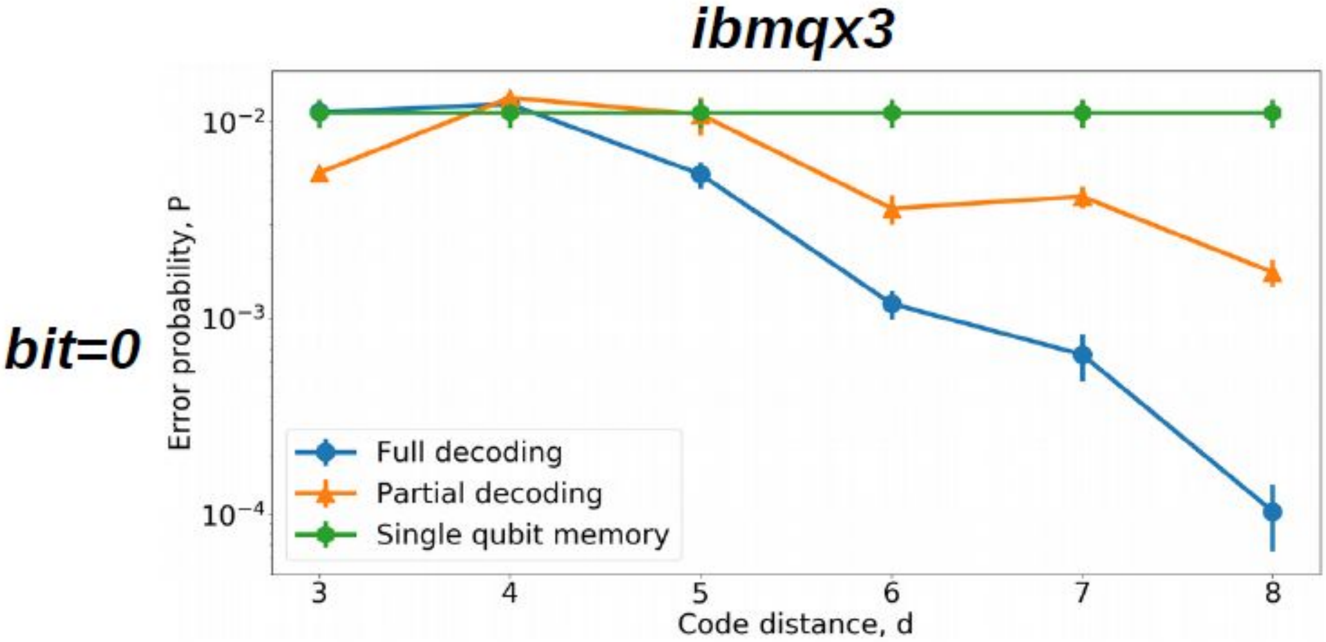
Quantum error-correction on IBM Q - state of the art

A repetition code of up to 15 qubits

- In [3], repetition codes of the distance *d* between 3 and 8 were studied.
- In this approach, the original qubit's state is repeated across *d* qubits, and ancilla qubits are used for storing information about the parity of pairs of qubits.
- The decoding procedure uses lookup tables filled with experimental data, in compliance with the *majority voting* method.
- Results showed that the error rate decays exponentially with the code distance.
- Results showed that decoding based on both code and ancilla qubits gives better results, compared to decoding based on code qubits only.

[3] J. R. Wootton and D. Loss, "Repetition code of 15 qubits," Physical Review A, vol. 97, no. 5, May 2018.

A repetition code of up to 15 qubits - results



Source: [3]

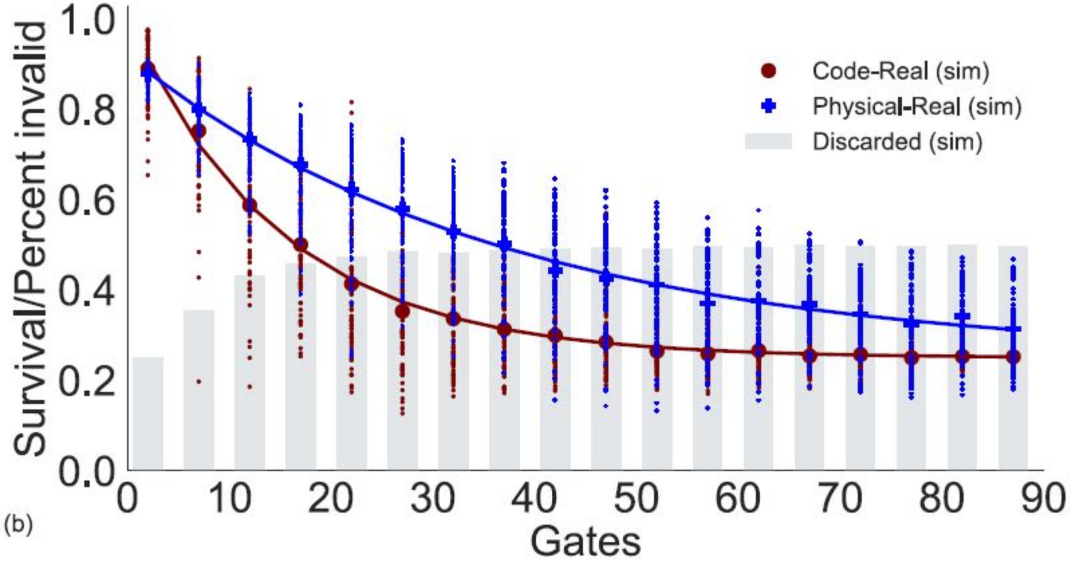
[3] J. R. Wootton and D. Loss, “Repetition code of 15 qubits,” Physical Review A, vol. 97, no. 5, May 2018.

Implementations of [[4, 2, 2]] QECC

- In [8], it was stated that the [[4, 2, 2]] code, which can be implemented with merely 5 qubits, can be used to improve a quantum circuit's fault-tolerance.
- Implementations of [[4, 2, 2]] codes on IBM Q devices were proposed in quite many papers, e. g. [4, 5, 6, 7].
- [[4, 2, 2]] can only detect errors. In [4, 5, 6], a post-selection method is used to discard results produced by a faulty computation.
- The randomized benchmarking protocol was used in [5, 7].
- Tests showed that the computation infidelity can be diminished with the use of [[4, 2, 2]] code.

[4] J. Roffe, D. Headley, N. Chancellor, D. Horsman, and V. Kendon, "Protecting quantum memories using coherent parity check codes," Quantum Science and Technology. 3. 10.1088/2058-9565/aac64e, May 2018.
[5] R. Harper and S. T. Flammia, "Fault-Tolerant Logical Gates in the IBM Quantum Experience," Physical Review Letters, vol. 122, no. 8, Feb. 2019.
[6] A. Hu and J. Li and R. Shapiro, "Quantum Benchmarking on the [[4, 2, 2]] Code," 2018.
[7] C. Vuillot, "Is error detection helpful on IBM 5Q chips ?," Quantum information & computation. 18. 0949-, 2018.
[8] D. Gottesman, "Quantum fault tolerance in small experiments," 2016.

Implementations of [[4, 2, 2]] QECC - results



Source: [5]

[5] R. Harper and S. T. Flammia, “Fault-Tolerant Logical Gates in the IBM Quantum Experience,” Physical Review Letters, vol. 122, no. 8, Feb. 2019.

Nondestructive discrimination and automated error-correction for Bell and GHZ states

- In [9], a procedure for non-destructive discrimination and automated error-correction for Bell states was proposed.
- In this approach, repetition of the original state is not used, but additional information needed for error detection and correction is projected to and obtained from ancilla qubits.
- Experimental realizations of the procedure using a 5-qubit IBM Q computer can be found in [10, 11].
- In [11], an analogous procedure for GHZ states was realized as well.
- The results showed that the procedures improve the output state’s fidelity, however the more complex the circuit, the worse the results.

[9] P. Pandey, E. Sriram Prasath, M. Gupta and P. Panigrahi, “Automated Error Correction For Generalized Bell States,” 2010.
[10] M. Sisodia, A. Shukla and A. Pathak, “Experimental realization of nondestructive discrimination of Bell states using a five-qubit quantum computer,” Physics Letters A. 381. 10.1016/j.physleta.2017.09.050, 2017.
[11] D. Ghosh, P. Agarwal, P. Pandey, B. K. Behera, and P. K. Panigrahi, “Automated error correction in IBM quantum computer and explicit generalization,” Quantum Information Processing, vol. 17, no. 6, May 2018.

Qiskit Ignis

- Qiskit Ignis [12] is a part of the Qiskit framework, aimed at analysing and mitigating the noise in the IBM Q devices.
- Among all, it provides:
  - implementation of repetition codes,
  - methods for benchmarking quantum error-correcting codes.

[12] “Qiskit Ignis,” https://qiskit.org/ignis/.

A new implementation of the five-qubit code using Qiskit

The five-qubit code

- It's a [[5, 1, 3]] quantum error-correcting code.
- The smallest qecc that corrects one, arbitrary single-qubit error.
- Four measurements with binary outputs - 16 possible error syndromes.
- 15 possible errors (3 error types: X, Y, Z; 5 code qubits) and one correct state.
- Each error syndrome indicates a specific error.
- Example of a stabilizer code [13].

[13] D. Gottesman, " Stabilizer Codes and Quantum Error Correction," 1997.

Five-qubit qecc, encoding

- The new, logical base states are:

$$\begin{aligned}
|\overline{0}\rangle \;=\;& |00000\rangle + |10010\rangle + |01001\rangle + |10100\rangle \\
& + |01010\rangle - |11011\rangle - |00110\rangle - |11000\rangle \\
& - |11101\rangle - |00011\rangle - |11110\rangle - |01111\rangle \\
& - |10001\rangle - |01100\rangle - |10111\rangle + |00101\rangle,
\end{aligned}$$

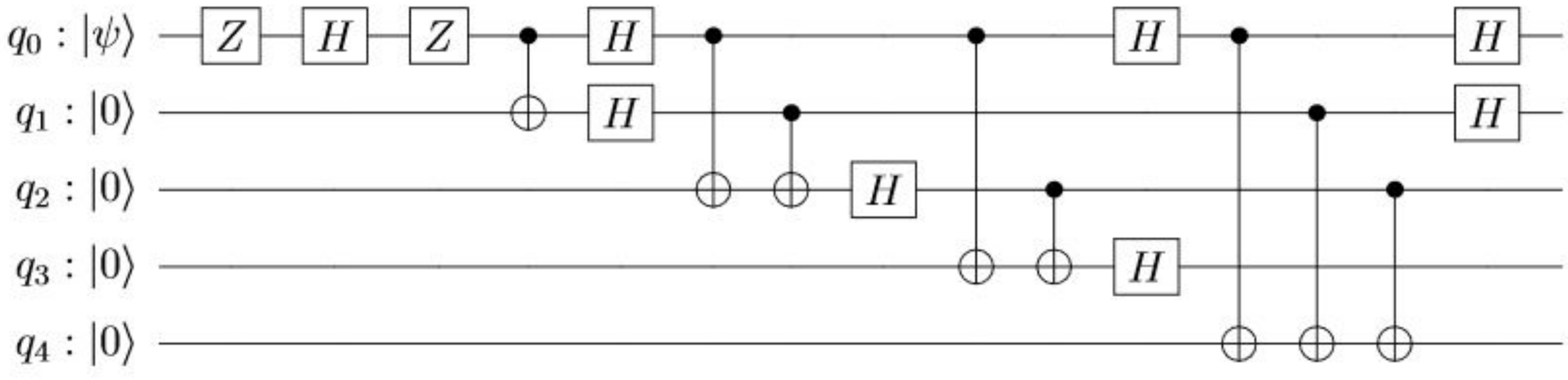
$$\begin{aligned}
|\overline{1}\rangle \;=\;& |11111\rangle + |01101\rangle + |10110\rangle + |01011\rangle \\
& + |10101\rangle - |00100\rangle - |11001\rangle - |00111\rangle \\
& - |00010\rangle - |11100\rangle - |00001\rangle - |10000\rangle \\
& - |01110\rangle - |10011\rangle - |01000\rangle + |11010\rangle.
\end{aligned}$$

Source: [13]

[13] D. Gottesman, “ Stabilizer Codes and Quantum Error Correction,” 1997.

Five-qubit qecc, encoding cont.

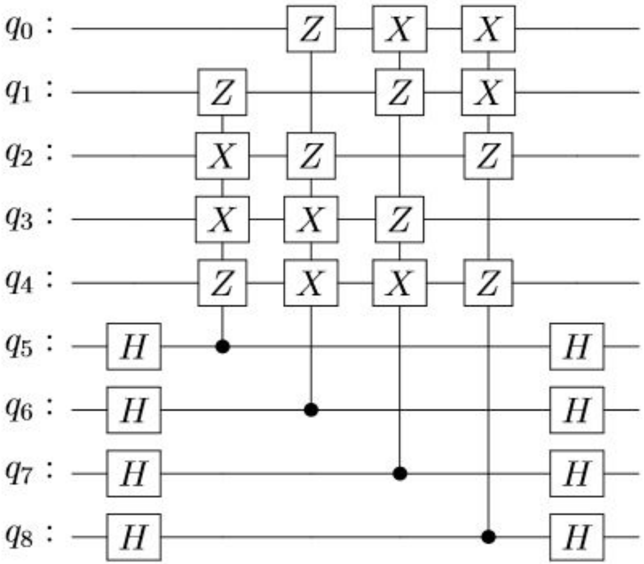
- Encoding procedure for the five-qubit code, proposed in [14].



[14] N. David Mermin, “Lecture Notes on Quantum Computation,” Chapter 5, Cornell University, Physics 481-681, CS 483; Spring, 2006

Five-qubit qecc, syndrome measurement

- Syndrome measurement procedure for the five-qubit code.



Five-qubit qecc, syndrome measurement interpretation

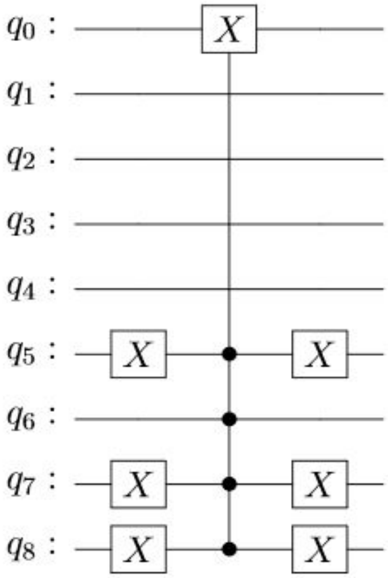
- Each of the possible error syndromes indicates either a specific error or a correct state.

	$X_0Y_0Z_0$	$X_1Y_1Z_1$	$X_2Y_2Z_2$	$X_3Y_3Z_3$	$X_4Y_4Z_4$	1
$M_0 = Z_1X_2X_3Z_4$	$+++$	$--+$	$+--$	$+--$	$--+$	$+$
$M_1 = Z_2X_3X_4Z_0$	$--+$	$+++$	$--+$	$+--$	$+--$	$+$
$M_2 = Z_3X_4X_0Z_1$	$+--$	$--+$	$+++$	$--+$	$+--$	$+$
$M_3 = Z_4X_0X_1Z_2$	$+--$	$+--$	$--+$	$+++$	$--+$	$+$

Source: [12]

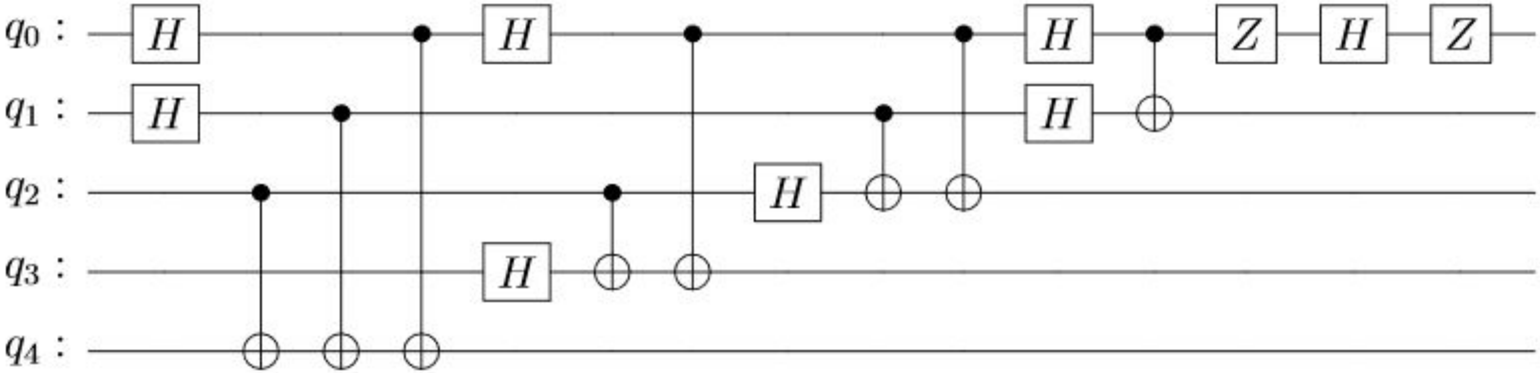
Five-qubit qecc, error-correction

- Error correction is performed for each error syndrome separately.
- Example: correction of a bit flip on the qubit 0.



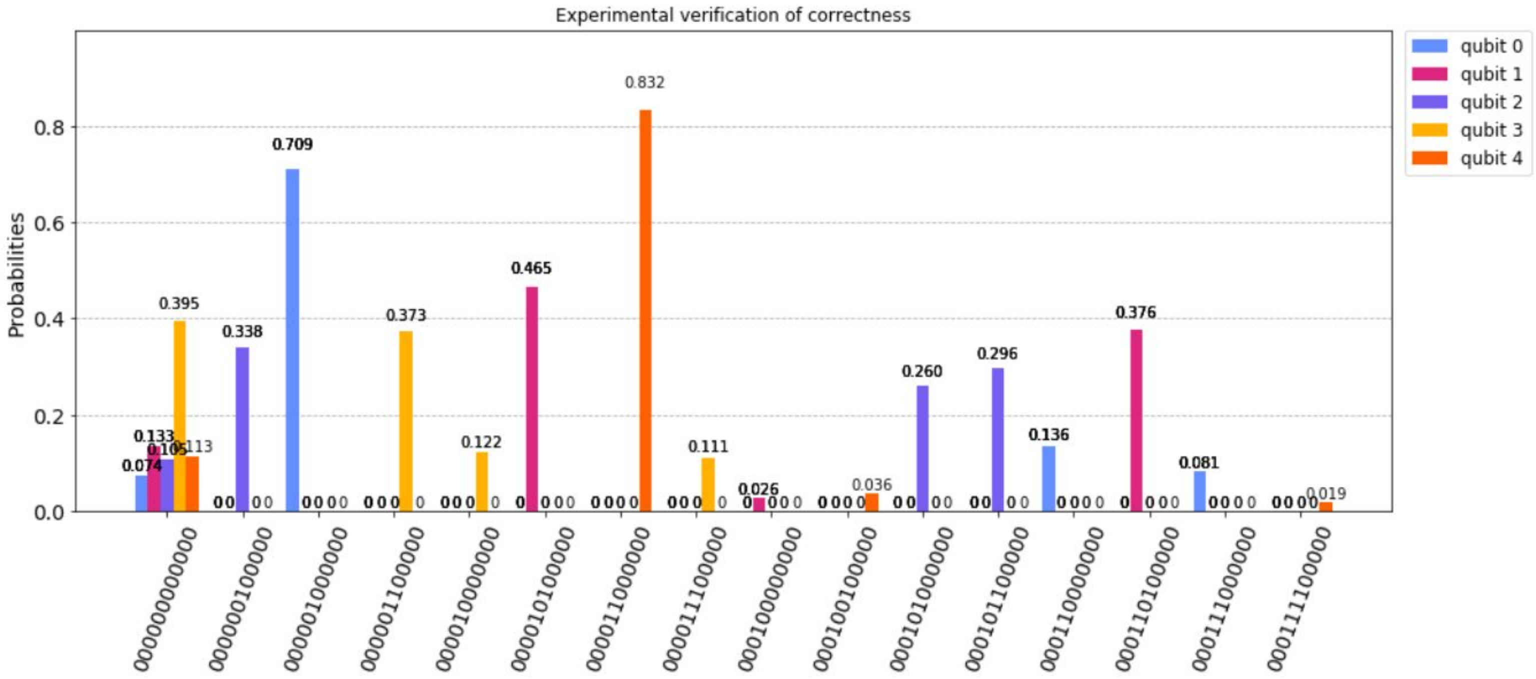
Five-qubit qecc, decoding

- Decoding is performed.



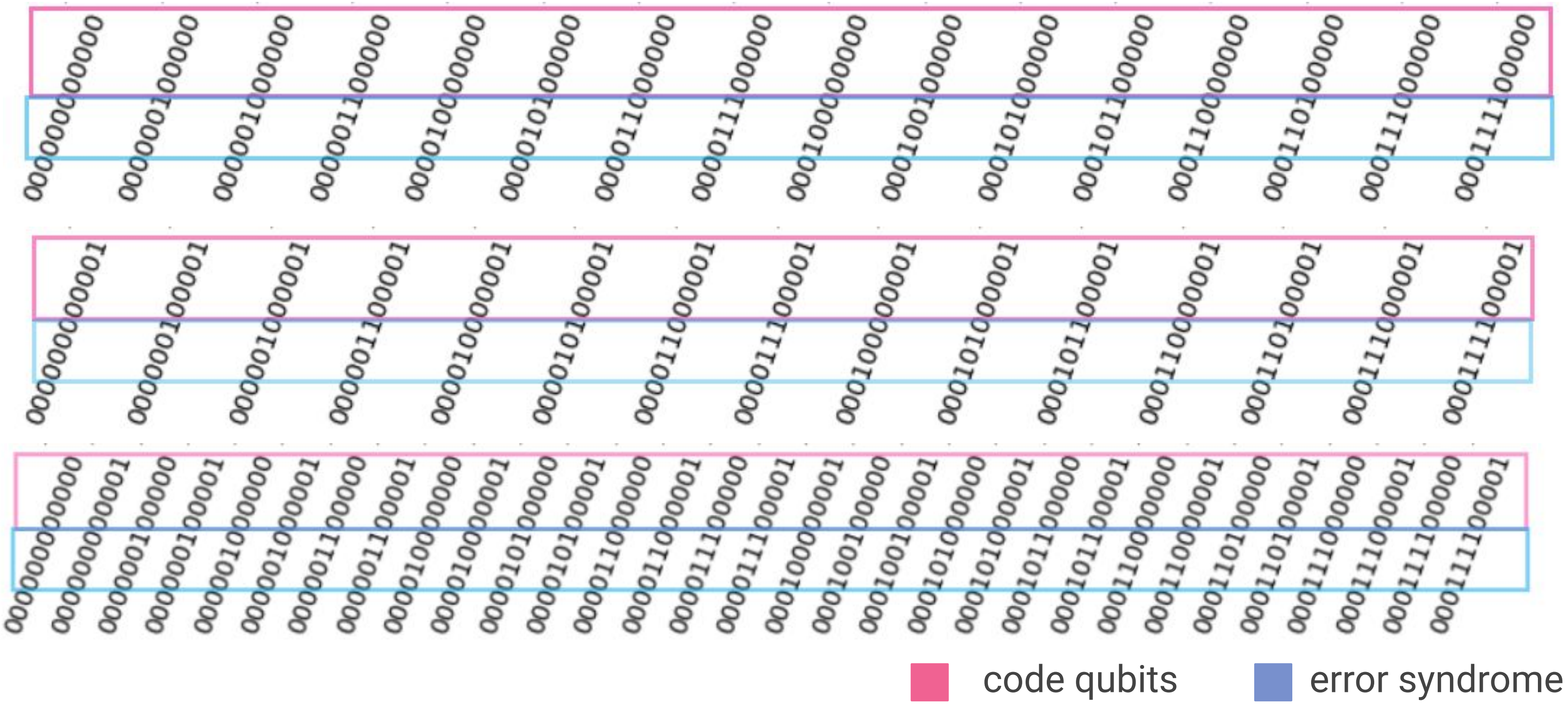
Experimental verification of correctness

- Experiment was run on the ibmq_qasm_simulator.
- Random, unitary single-qubit gates were applied to the code qubits.
- Initial state always remains untouched, and error syndromes can be observed on the ancilla qubits.



Experimental verification of correctness cont.

- Outputs for initial states |0>, |1> and an equal superposition of |0> and |1>.



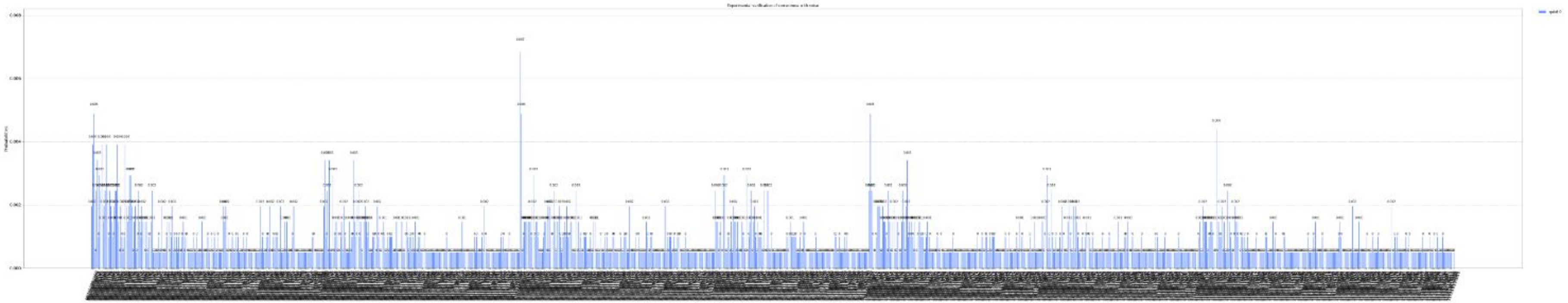
Applicability on IBM Q devices

- The five-qubit qecc implementation uses 12 qubits.
- The only real quantum IBM Q device with a sufficient number of qubits is the ibmq_16_melbourne, which has 15 qubits.
- However, it occurred that the circuit consists of too many quantum operations, and cannot be run on this device.
- Solution 1: the error-correction procedure can be optimized to lower the number of used gates, especially the Toffoli gates - future work.
- Solution 2: The environment of the ibmq_16_melbourne can be simulated on the ibmq_qasm_simulator, with the use of appropriate noise model [14], coupling map and instruction set.

[14] https://qiskit.org/documentation/stubs/qiskit.providers.aer.noise.NoiseModel.html

Results of simulation of the ibmq_16_melbourne device

- Experimental verification of correctness was run on the ibmq_qasm_simulator, with the noise model , coupling map and instruction set of ibmq_16_melbourne.
- The results are much worse compared to the experiment run on the simulator without noise. It can be observed, that uncorrect states are often measured.



Results of simulation of the ibmq_16_melbourne device

- Problem: the circuit for the five-qubit code itself consists of a lot of quantum gates.
    - Solution: optimizing the error-correction procedure might help in this case as well.
- Problem 2: errors occur not only on the code qubits, but also on the ancilla qubits.
    - Solution: performing tests with noise introduced only tho the code qubits - future work.

Summary and future work

Summary

- Errors in quantum computers have to be dealt with to enable reliable quantum computations.
- The use of quantum error-correcting codes can improve the fault-tolerance of quantum computations.
- Quantum error-correcting codes can be successfully implemented with the use of Qiskit framework.
- Currently available IBM Q devices are not adapted for complex quantum programs. Not only the number of qubits, but also the number of gates that can be applied is limited.
- With the development of quantum devices, the usefulness quantum error-correcting code will rise.

Future work

- Testing the efficacy of the five-qubit code, e. g with the randomized benchmarking procedure.
- Combining the use of quantum error-correcting codes with an adaption of the circuit to the particular architecture of the target quantum device.
- Implementation of new quantum error-correcting codes, e. g. the seven-qubit code.

Thank you